Going the Last Mile: Access Regulation and Vertical Integration
Preliminary Version

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Abstract

In many markets entry requires a significant infrastructure investment which can lead to inefficiently low competition and even monopolies in many cases. One solution adopted by many countries is to require the owner of this infrastructure to allow competitors to rent access at a regulated price. In this case the network owner becomes a wholesale provider of infrastructure services who is also participating in the retail market. Another solution is to separate the network owner into a wholesale firm and a vertically separate retail firm. This paper compares infrastructure quality investment incentives for the network owner under these two regimes. Retail prices will be higher under the vertically separated regime, meaning that quality investment will attract more consumers with a separated firm, but the ability to participate on the retail market in addition to the heavily regulated wholesale market means that a vertically integrated owner will have more incentive to invest when there is significant horizontal differentiation between retail firms.

In many markets such as electricity, rail transport, and internet service provision, entry requires a significant infrastructure investment which can lead to inefficiently low competition and even monopolies in many cases. In the case of internet service provision, connecting a central exchange to the internet backbone is relatively cheap, but connecting central exchanges to individual homes (known as "last mile" infrastructure) can be prohibitively expensive. In the United States 50% of households have access to one *or fewer* high speed Internet providers, and the price per megabit is much higher than in other developed countries (Federal Communications Commission 2017). One possible solution to this problem is to require owners of vital infrastructure to license it out to competitors on the retail market at a regulated price. Ofcom (the United Kingdom's equivalent to the FCC) requires British Telecom (BT) to rent out its last mile infrastructure to competitors in exchange for a per-household yearly line rental fee[1].

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1. As a result, the average British Citizen has access to between 3 and 4 options for internet provision, and prices tend to be lower on a per-megabit basis than in the United States (Federal Communications Commission 2017; Ofcom 2016; Nardotto, Valletti, and Verboven 2015).

BT, and other incumbents in similar markets often argue that this forced rental process (often referred to as "unbundling") reduces their incentive to invest because it diverts much of the benefit from investment to the retail firms which rent its infrastructure. They claim that while prices will be cheaper, it comes at the cost of lower quality service for consumers. On the other hand, market entrants taking advantage of unbundling often argue that because the regulation increases competition, it forces the network owner to remain maintain a higher quality standard than they would otherwise if they were not being disciplined by market forces.

While BT's investment has lagged behind other network firms in the United Kingdom, the main reason for this appears to be selective investment in areas where BT has a large number of customers and lack of maintenance where most consumers on its network choose other firms. As a result of this behavior Ofcom has legally separated BT's ISP division and Openreach (Ofcom 2017), which was previously the wing of BT responsible for providing last mile network services. Whereas previously the market in these regions contained a set of competing retail ISPs and one retail ISP which was vertically integrated with the wholesale network infrastructure owner, all ISP retailers operating on BT's infrastructure are now vertically separate from the wholesaler. The purpose of this paper is to examine how changing the vertical structure of a network owner affects quality investment in the context of a market with network access regulation.

A network owner who participates in the retail market will gain direct benefits from increasing quality because they will be able to increase their retail price, as well as seeing increased sales in both its direct and wholesale businesses. Additionally, this integrated firm has a cost advantage over other retailers because it does not need to pay the rental fee to access the network, and so market retail prices will be lower. Because the retail price is set by the government a vertically separated network owner cannot capture as much of the benefit from investment, but the increased retail prices mean that more consumers will choose the outside option for any given quality level, so the separated network owner may wish to invest more in quality to increase total market demand. My findings can be summarized as follows:

1. The market equilibrium with an integrated firm will have lower prices and as a result demand will be less sensitive to a change in quality than under a vertically separated regime.

    - In the case of high retail prices, the marginal revenue to the network owner from increasing quality will increase with vertical integration even though the number of marginal consumers who come on to the network decreases. High prices tend to be the result of horizontal differentiation so a vertically integrated network owner in a highly differentiated market will invest more than a separated network owner
    - If the retail firms do not have much horizontal differentiation, then retail prices will be relatively low, and the vertically separated network owner will have a stronger investment incentive.

2. An increase in quality will lead to a concomitant increase in prices, but the increase in prices will always be less than the increase in utility to consumers, meaning that more quality investment will always be better for consumers.

3. Because prices represent transfers between agents participating in the market, prices only affect total welfare by changing behavior. However the integrated retailer directly benefits from being able to increase prices, and so an integrated network owner may wish to over-invest in quality from the view of social optimality. Accordingly, a regulator may prefer a vertically separated regime despite lower quality investment.

The rest of the paper is organized as follows: section 1 goes over the related literature, section 2 describes the model, conditions for equilibrium, and results, and section 3 describes my plans for future additions to the paper.

1 Related Literature

There is a rich literature exploring the effects of access regulation on welfare and investment incentives. Laffont and Tirole (1994) provide a good introduction to the conceptual framework, and Laffont and Tirole (2001) is a nice overview of some of the early literature, most of which focuses on comparing a market with regulation to one without or on the effects of changing the rental price. Nardotto, Valletti, and Verboven (2015) show that unbundling initially increases investment in growing markets, but will have little effect on investment in mature markets with established firms. This aligns with the results in this paper, as the primary benefit of quality investment at the network level is decreasing the appeal of the outside option. If most consumers are already participating in the network, then the incentive to invest is already low and regime changes would not have much room to create siginificant shifts.

Guthrie (2006) surveys various results on the effects of various regulatory structures on infrastructure investment, and finds that the effects of access regulation depend on how much quality of service relies on the network owner's investment and how much on the entrant's own capital. If quality is entirely dependent on the owner, then increased competition will reduce the owner's returns to investment, but if a significant portion of quality depends on the individual firms, then competition will lead to increased investment. Vareda (2007) is the paper most similar to this one in the access pricing literature in terms of structure in the retail market, although he asks a different research question. He uses a setting with a Hotelling-style duopoly in the final goods market and both quality and cost reducing investment on the part of the network owner. He finds that a lower access price reduces the incentive to invest in quality, but increases the incentive to reduce costs if the regulator can commit to a fixed access charge. These findings suggest that the results of this paper may depend on the fact that quality is entirely a decision of the network owner. If there were an individual component to investment, then these the competitive advantage of the vertically integrated firm would lead it to increase individual investment compared to an unintegrated baseline, but the competing retail firms, being disadvantaged, would invest less.

Foros (2004) considers a market similar to the one in this paper, but is more interested in the relationship of quality investment to access pricing. He considers a market with a duopoly in the final goods market and firms with differing (exogenously imposed) quality increases as a result of quality investment. He finds that the effect of access regulation on quality investment depends on the relative ability of firms to take advantage of quality increases. If the network owner has equal or better advantage from quality investment, then the owner will set a high wholesale price in order to prevent the competitor from accessing the market. Access price regulation will increase investment if the owner has greater benefit from investment because the owner will use investment to widen the quality difference between firms to drive a competitive advantage. If the competitor firm has greater benefit from quality, then access price regulation reduces quality investment and total welfare, but the regulation will not be necessary because the network owner will set a (still high) wholesale price that allows the competitor onto the market anyway in order to take advantage of the competitors greater ability to attract customers. My result that investment is not strictly increasing (and often non-monotonic) in the regulated access price matches this result, but I derive a competitive advantage endogenously rather than imposing it as a parameter in the model.

The result that an integrated firm will result in lower market prices than having a vertically separated network owner firm is consistent with much of the vertical integration and double marginalization literature going as far back as Cournot (1927). Chiang, Chhajed, and Hess (2003) and Tsay and Agrawal (2004) both show that an equilibrium with an integrated firm can have both lower prices and more profits for both the end firms and the supplier compared to an equilibrium with strict vertical separation. The primary reason given in the literature is that the integrated firm has a greater marginal benefit from a price decrease, and the presence of the integrated firm can help with supplier/retailer wholesaler coordination, but the reason in this paper is that the integrated firm has a cost advantage. The regulator sets the wholesale price, so the price bump is not a double marginalization problem despite having the flavor of one[2]. This is because the integrated firm has a cost advantage in the final goods market, but since it makes money from wholesale provision as well as final good sales it uses this power to soften competition rather than dominate the market[3].

There are a number of vertical integration papers that consider quality choice. Economides (1999) is probably the best known one, and his result is that a vertically integrated firm will always choose to invest in higher quality. The reason for this is exactly the same reason that an integrated firm chooses a lower retail price in Cournot (1927): the direct benefits accrued by an integrated firm are significantly greater than the filtered benefits a wholesaler receives. In the case of Cournot, this means that it can set a retail price above the wholesale price but below the vertically separated equilibrium retail price, resulting in a pareto improvement for both the firm and consumers. One of the main differences that produces ambiguity in my result where Cournot and Economides both find that integration is unambiguously superior is that in my setting the wholesale price is regulated, thus eliminating the double marginalization effect that reduces welfare under separation.

Buehler, Schmutzler, and Benz (2004) find that a vertically separated integrated firm may have higher investment incentives than a vertically integrated firm, but via a different mechanism than in this paper. In their framework, investment is higher with separation only if a quality increase will cause lower prices and increased demand. In that case the quality increase is used to counteract the price increase from vertical separation. In this paper a quality increase will always increase both price and demand, but the reason that the network owner invests more is again counteracting this increase in prices under vertical separation.

2. Chiang, Chhajed, and Hess (2003) briefly touch on costs in their paper

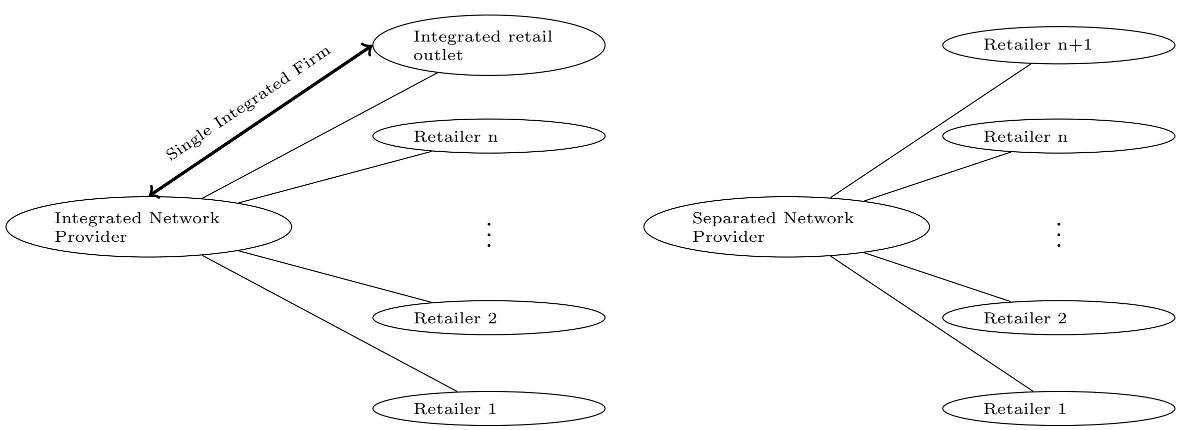
3. See Arya, Mittendorf, and Sappington (2008) for a more thorough exploration of this phenomenon

The differences between their results and mine are due partly to a simplified market structure (they compare vertical integration with serial monopolies, whereas this paper allows for a richer competitive setting) and partly due to a reduced form demand function leading to an ambiguous equilibrium response to an increase in quality[4].

González and Kujal (2012) consider vertical integration with competition in the retail and upstream markets. They find that vertical integration in this setting will always increase quality investment, although their model differs from mine in that quality is a decision of the retail firm. If one of the firms is vertically integrated, then the combination of reduced double marginalization and its cost advantage will give it a competitive edge. The retail firm will then invest in higher quality to widen this advantage over its competitors.

2 The Model

The agents in the model consist of a regulator whose goal is to maximize social welfare, a continuum of consumers with total mass of 1, a firm which owns an excludable input which is necessary for provision of goods in the final market, and n ($n+1$ when the network owner is vertically separated) competing retail firms who have the right to access the input at a rental price $r \geq 0$ per customer. The the owner firm is denoted by o, and the competing retail firms are indexed by the subscript j. The regulator chooses whether to allow o to remain as an integrated firm or to split it into a network wholesaler and a retail provider. Each firm $i \in \{j\}_{j=1}^{n} \cup \{o\}$ or $i \in \{j\}_{j=1}^{n+1}$ (depending on whether o is vertically integrated) chooses a final goods price, and o additionally chooses an investment level q for the quality of the final good. The structure of the market is shown in fig. 1.



(a) Vertical integration (b) Vertical separation

Figure 1: The market structure in the integrated and separated cases.

The most important point to emphasize is that the number of firms participating in the

4. Buehler, Schmutzler, and Benz use a demand function of the form $D(p, q)$ where p is price and q is quality. Demand is assumed decreasing in price and increasing in quality, with appropriate concavities to ensure equilibrium. Because part of my interest is the effect of competition, I use a random utility framework in the vein of Quint (2014). As a result of their more general framework, either demand or price can reduce as a result of an increase in quality. They cannot both go down but it is possible for both to go up.

retail market is constant. This is to ensure that all of the differences we see between the two regimes come from the ownership structure and not from changes in competition.

The order of the game is shown in fig. 2, but can be summarized as follows

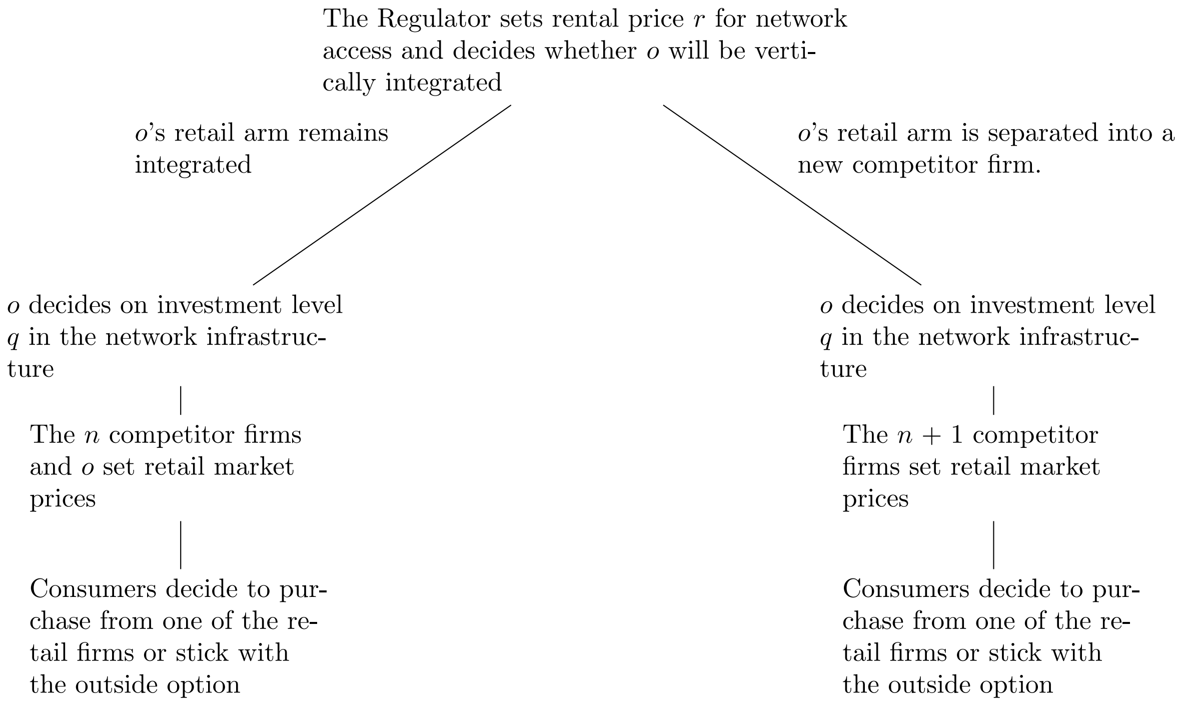


Figure 2: The order of actions for agents in this game.

1. The regulator sets rental price r for the infrastructure and decides whether o will be vertically integrated.
2. The network owner firm o decides on investment level q in the infrastructure[5]
3. Consumers and the firms on the retail market participate in a retail market game. Firm competition is based on price.

The solution concept is sub-game perfect equilibrium, and I proceed via backward induction.

Consumers

Consumers have quasi-linear utility given by

$$u(q) - p_i + \epsilon_{mi}$$

Where $u(q), q \in [0, \infty)$ is a concave increasing function of quality following the Inada conditions $lim_{q \to 0}\, u'(q) = \infty$, and $lim_{q \to \infty}\, u'(q) = 0$. I assume that $u(q)$ is twice differentiable

5. if o is vertically separated, then we will continue to use o to denote the network owner and the new separate final goods firm will treated as another competitor firm.

for $q > 0$. Quality will depend only on o's investment in the network, and so it will be common across all firms. p_i is the final good price for firm i, and ϵ_{mi} is a randomly drawn horizontal differentiation term for consumer m at firm i with pdf $f(\epsilon)$. Consumers also have access to an outside option labeled θ with price 0, quality 0 and utility draw ϵ_θ. I make following assumptions

Assumption 1. *$f(\cdot)$ is unbounded, strictly log concave, and atomless.*

Assumption 2. *ϵ_{mi} is i.i.d. for all m and i.*

Assumption 1 is a set of regularity assumptions that ensure existence of equilibrium as well as ensuring that demand will be continuous. Log concavity on the horizontal differentiation draws will give quasi-concavity of the profit functions in retail prices (although more is needed in the case of the vertically integrated retailer, see assumption 3 below). Assumption 2 is slightly more restrictive than assumption 1, but it will ensure that the un-integrated retailers behave symmetrically. This ensures that any differences in behavior across market structures come from the change in regime and are not an artifact of asymmetry in the retail firm.

Consumers have unit demand and will purchase from the option which provides maximal utility, i.e. the i such that

$$u(q) - p_i + \epsilon_{mi} \geq u(q) - p_k + \epsilon_{mk} \wedge u(q) - p_i + \epsilon_{mi} > \epsilon_{m\theta} \ \forall k \tag{1}$$

In order to simplify notation, I define

$$\Delta_{ik} = p_k - p_i, \quad \Delta_{i\theta} = u(q) - p_i$$

Δ_{ik} will be positive if $p_i < p_k$, and so it represents firm i's price advantage over firm k. Similarly, $u(q)$ is the effective price of the outside option, so $\Delta_{i\theta}$ will represent i's effective price advantage over the outside option. With this notation we can rewrite eq. (1) as

$$\epsilon_{mk} \leq \epsilon_{mi} + \Delta_{ik} \wedge \epsilon_{m\theta} \leq \epsilon_{mi} + \Delta_{i\theta} \ \forall \ k \neq i$$

Letting P be the vector of prices, denote demand for firm i by $d_i(P, q)$. Since consumers are ex ante identical and have total mass 1, this demand will simply be the probability that eq. (1) is true. The ϵ draws are independent, and the set of consumers who are indifferent between any two options will have measure 0 so for a given epsilon, $Pr(\epsilon_k < \epsilon_i + \Delta_{ik} \ \forall k) = \prod_{k \neq i} F(\epsilon + \Delta_{ik})$, and $Pr(\epsilon_\theta < \epsilon_i + \Delta_{\theta k}) = F(\epsilon + \Delta_{i\theta})$. By unboundedness of $f(\cdot)$ this event can occur for any ϵ_i so we multiply these probabilities together and integrate across $f(\epsilon)$ to get demand

$$d_i(P, q) = \int_{-\infty}^{\infty} f(\epsilon) F(\epsilon + \Delta_{i\theta}) \prod_{k \neq i} F(\epsilon + \Delta_{ik}) d\epsilon \tag{2}$$

and the mass of consumers choosing the outside option is the probability that the outside option gives the most utility. It is derived analagously to firm demand

$$d_\theta(P, q) = \int_{-\infty}^{\infty} f(\epsilon) \prod_{k} F(\epsilon - \Delta_{k\theta}) d\epsilon \tag{3}$$

When firm j increases its price, the marginal consumers who choose to leave that firm will then switch to the choice which was previously their second most preferred option. The relative appeal of the other options to each other is unchanged, thus the increase in demand for firm $i \neq j$ must be some proportion of the decrease in demand for firm j. This intuition is formalized in lemma 1.

Lemma 1. *Under assumption 1 and assumption 2*

$$\frac{\partial d_i(P,q)}{\partial p_j} = -s_i^j(P,q)\frac{\partial d_j(P,q)}{\partial p_j}$$

Where $s_i^j(P,q) = \frac{\frac{\partial d_i(P,q)}{\partial p_j}}{\frac{\partial d_\theta(P,q)}{\partial p_j} + \sum_{i \neq j} \frac{\partial d_i(P,q)}{\partial p_j}} < 1$

Proof. See appendix A.1 ■

The relative proportion of consumers who are indifferent between the network owner and one of the competitor firms and consumers who are indifferent between the network owner and the outside option will play a crucial role in the owner's pricing and investment decisions when it is vertically integrated. When an integrated retail firm lowers its price, this will attract new customers, but $1 - s_\theta^o(P,q)$ of these customers will arrive from the competing retailers, meaning that attracting these consumers comes at an opportunity cost of $1 - s_\theta^o(P,q)$ times the rental price. Additionally, if the owner increases quality then more consumers will come on to the network. The marginal revenue of these additional consumers to the network owner is the weighted average of the integrated firm's retail price and the rental price, where the weights are determined by the share marginal consumers who choose the integrated retailer.

Un-Integrated Retailers

The un-integrated retailers' profits will be the margin between their price and the rental price times their demand.

$$V_j = (p_j - r)d_j(P,q) \tag{4}$$

To ensure existence of equilibrium I restrict prices to $[0, \bar{P}(q,r)]$, but $\bar{P}(q,r)$ will be sufficiently large that no firm will end up at a corner[6] so this restriction is a purely technical assumption. Since the competitor firms pay the access fee r on a per-customer basis, it acts as a marginal cost of production. I that the actual cost of production is constant and

6. This upper bound may depend on q and r. See the proof of proposition 1 for a detailed discussion of why no firm will optimally price at a corner.

normalize it to 0. Given the symmetry of the un-integrated retailers, I can use Lemma 1 in Quint (2014) to show that these firms will choose the same price in equilibrium[7]. Accordingly

$$d_j(P,q) = \int_{-\infty}^{\infty} f(\epsilon)F(\epsilon + \Delta_{j\theta})F(\epsilon - \Delta_{oj})F(\epsilon)^{n-1}d\epsilon$$

Substituting eq. (2) into eq. (4) and taking the first order condition with regard to a change in price, then imposing symmetry, it is easy to derive that

$$p_j = r - \left(\frac{\partial \ln(d_j(P,q))}{\partial p_j}\right)^{-1}$$

under vertical integration this expands to

$$p_j = r + \frac{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon + \Delta_{j\theta})F(\epsilon - \Delta_{oj})F(\epsilon)^{n-1}d\epsilon}{\int_{-\infty}^{\infty} f(\epsilon)\Big[F(\epsilon + \Delta_{j\theta})(n-1)f(\epsilon)F(\epsilon)^{n-2}F(\epsilon - \Delta_{oj}) + F(\epsilon + \Delta_{j\theta})F(\epsilon)^{n-1}f(\epsilon - \Delta_{oj}) + f(\epsilon + \Delta_{j\theta})F(\epsilon - \Delta_{oj})F(\epsilon)^{n-1}\Big]d\epsilon} \tag{5}$$

and with vertical separation

$$p_j = r + \frac{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon + \Delta_{j\theta})F(\epsilon)^{n}d\epsilon}{\int_{-\infty}^{\infty} f(\epsilon)\Big[F(\epsilon + \Delta_{j\theta})nf(\epsilon)F(\epsilon)^{n-1} + f(\epsilon + \Delta_{j\theta})F(\epsilon)^{n}\Big]d\epsilon} \tag{6}$$

The first term on the right hand side of each equation is the retailers' effective marginal cost of production, which is the rental price r. The second term is the profit maximizing markup over this price. From Quint (2014), $d_i(P,q)$ is log concave in p_i for all i, so the second term on the right hand side will be decreasing in p_j [8]. Since the RHS will be above 0 when the LHS is 0, and the LHS is unbounded above, a solution is guaranteed to exist. Furthermore, log concavity of demand is sufficient to guarantee quasi-concavity of V_j in p_j, so the first order condition is sufficient for profit maximization.

Network Owner

If the government forces vertical separation, then the network owner has the value function

$$V_o = r\sum_j d_j(P,q) - \alpha(q)$$

The first term is the sum of the rental fees paid by the competitor firms. Since the fee is on a per-household basis, it will be scaled by market demand. $\alpha(q)$ is the cost of quality investment. It is an increasing, convex function following the Inada conditions $lim_{q\to 0}\,\alpha'(q) = 0$, and $lim_{q\to\infty}\,\alpha'(q) = \infty$., Without loss of generality I can assume $\alpha(0) = 0$.

7. From the Lemma price responses of the retailers will be supermodular. Symmetry follows immediately from symmetry of the first order conditions.

8. See Bagnoli and Bergstrom (2005) for a detailed summary of the properties of log concave functions.

If vertical integration is allowed, then

$$V_o = p_o d_o(P,q) + r \sum_j d_j(P,q) - \alpha(q)$$

The firm has profits from direct sales as well as the rental income and cost of investment. Since the number of final goods firms is constant in each regime, when the network owner is forced to vertically separate the new retail firm is symmetric with the other retail firms. So $\sum_j d_j(P,q)$ will contain n terms with vertical integration and $n+1$ terms under vertical separation. Taking the derivative of V_o with regard to p_o and finding the first order condition gives

$$p_o = nr\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}} + \frac{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{o\theta})F(\epsilon+\Delta_{oj})^n d\epsilon}{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{o\theta})f(\epsilon+\Delta_{oj})nF(\epsilon+\Delta_{oj})^{n-1} + f(\epsilon)f(\epsilon+\Delta_{o\theta})F(\epsilon+\Delta_{oj})^n d\epsilon} \tag{7}$$

This equation has the same cost/markup format of the FOC for the un-integrated retailers. Where $nr\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}}$ represents the lost rental fees due to stealing consumers from the other firms. Note that from lemma 1, $\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}} = s_j^o(P,q)$, so

$$nr\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}} = nrs_j^o(P,q) = r(1 - s_\theta^o(P,q))$$

as described above. Again, $d_o(P,q)$, will be log concave in p_o, so the markup term is decreasing in retail price, but if $s_j^o(P,q)$ increases in p_o then the first order condition might not be sufficient for profit maximization (Similar to a firm with increasing returns to scale). Lemma 2 describes some of the properties of $s_j^o(P,q)$ and assumption 3 will guarantee sufficiency of the first order condition for profit maximization. The details are discussed in the proof of proposition 1.

Lemma 2. *Under assumption 1 and assumption 2, $nrs_j^o(P,q)$ is*

(i) Increasing, constant or decreasing in p_o if $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)}$ is a monotonically decreasing, constant, or increasing function of ϵ respectively.

- *If $\frac{f(\epsilon)}{F(\epsilon)}$ is not constant over an interval of positive but finite length, then $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)}$ will be increasing if $\Delta_{j\theta} > 0$, constant if $\Delta_{j\theta} = 0$, and decreasing if $\Delta_{j\theta} < 0$*

(ii) Less than r

(iii) Converges to r as $n \to \infty$

(iv) Decreasing in p_j and increasing in q if $f(\cdot)$ is symmetric, $\Delta_{o\theta} > \Delta_{j\theta}$ and $\Delta_{o\theta} > 0$

Proof. See Appendix A.2 ■

Assumption 3 follows from lemma 2 and ensures existence of a price equilibrium.

Assumption 3. *To ensure equilibrium in the pricing subgame, I assume*

(i) $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)}$ *is constant in* ϵ *or* $u(0)$ *is sufficiently large that no firm will set a price above* $u(q)$. $\frac{f(\epsilon)}{F(\epsilon)}$ *is strictly decreasing or constant in* ϵ[9].

To ensure uniqueness I will sometimes further assume

(ii) $f(\cdot)$ *is symmetric*

Assumption 3 is restrictive, but not egregiously so. The first part implies that being in the market has a base level of appeal even if the network owner barely invests in the network. Given that the motivating example is internet service provision and the multitudinous benefits of having internet access this seems to be a reasonable restriction to impose. $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)}$ will be constant in ϵ for the logistic and double exponential distributions and $\frac{f(\epsilon)}{F(\epsilon)}$ is monotonically decreasing for many other commonly used distributions such as the normal. Part (i) of assumption 3 is sufficient to ensure that $\Delta_{o\theta} > \Delta_{j\theta} > 0$. I have established all I need to prove proposition 1.

Proposition 1. *Under assumptions 1, 2 and 3(i), for any given* q *and* r

1. *Equilibrium in the pricing subgame exists for both market structures. If 3(ii) holds then is unique*

2. *Final good prices will be lower with a vertically integrated firm and* p_o *will be the lowest final good price*

3. *For any given* q, $u(q) - p_j$ *will be larger with a vertically integrated network owner*

And when o *chooses quality,*

4. *Equilibrium prices will be increasing in* q *for both the integrated and vertically separated equilibrium but* $\frac{\partial p_j}{\partial q} < u'(q)$ *for all* j.

5. $u(q) - p_j$ *will be increasing in* q

6. *Prices may be higher or lower in the integrated equilibrium than the equilibrium with a vertically separated network owner, but if they are higher then* $u(q^{int}) - p_j^{int} > u(q^{sep}) - p_j^{sep}$ *where "int" denotes the integrated and "sep" the vertically separated subgame.*

Proof. See Appendix A.3

■

9. Since $f(\cdot)$ is log concave this quotient is non-increasing. This part of the assumption rules out the possibility that $\frac{f(\epsilon)}{F(\epsilon)}$ is constant over a finite interval

Existence

Existence of equilibrium under vertical separation in 1 is a well established result (see Quint (2014) or Anderson, De Palma, and Nesterov (1995) for similar examples), but the structure of the network owner's revenue with vertical integration means that is necessary to establish quasi-concavity of the owner's profit in its own price before we can apply any existence theorems. We can see from eq. (7) that o faces an "opportunity cost of production" in that $1 - s_j^o(P, q)$ of the customers it attracts from the other retail firms, which means that o is giving up the rental fees that the these firms would have paid for those customers. Lemma 2 and assumption 3 ensure that this opportunity cost is decreasing in p_o, which is structurally equivalent to an increasing cost of production. Quasi-concavity follows from this result with relative ease.

Price Comparison

The decrease in prices with part 2 comes from the fact that the integrated firm's opportunity cost of attracting consumers is less than r, meaning that the owner has a cost advantage when providing retail services and will set a lower price than the other firms, who in turn respond by setting a lower price than they would in the vertically separated equilibrium. 3 follows immediately from 2 and shows that for fixed q, consumers will be better off if the network owner is vertically integrated.

The Price-Quality Relationship

4 and 5 show that while prices are increasing in quality, the firms are not able to capture all of the increase and so consumers will be better off as quality increases. Finally, point 6 acknowledges that while prices are lower under vertical integration for fixed quality, the vertically integrated firm may invest more than the separated firm, leading to higher prices and more utility for consumers.

Quality Choice

When choosing an investment level, the network owner balances the additional revenue driven by consumers attracted to the network by increased quality against the cost of investment.

Proposition 2. *Demand for all firms increases more in response to changes in quality when o is vertically separated. There exists a cutoff σ^* such that equilibrium quality investment is higher with the vertically integrated firm if $var(\epsilon_i) \geq \sigma^*$. Otherwise the comparison is ambiguous.*

Proof. See Appendix A.4

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With a vertically separated firm, all revenue comes from other firms, but there will be $n + 1$ such firms. With vertical integration, there are only n firms paying the rental fee, but the network owner will receive some additional revenue from consumers purchasing directly from it. The comparison in quality investment will then depend on three key factors:

1. The marginal rate at which consumers leave the outside option as a result of a quality increase.

2. The expected revenue from an additional consumer leaving the outside option as a result of the quality increase.

3. The competitor firms choosing a higher price in equilibrium as a result of the change in quality mitigating the demand increase from 1.

Marginal Demand Response

Since $u(q)$ is effectively the price of the outside option, theorem 1 in Quint (2014) gives that demand is more quality elastic in the equilibrium with vertical separation due to the lower equilibrium price. However, while the *percentage* change in demand for the outside option as a result of a quality increase is larger under vertical integration, the *magnitude* of the change is larger with the higher prices in vertical separation. The reason for this is that with lower prices under vertical integration, the consumers who received a low ϵ draw for the outside option have already left it, so an increase in its price doesn't affect their decision and the remaining consumers are less price sensitive and are thus less likely to change behavior in response to a quality increase.

Marginal Revenue

While fewer consumers are leaving the outside option, the marginal revenue for each consumer may be larger with vertical integration. This marginal revenue is r with a vertically separated network owner and $rns_j^\theta(P,q) + p_o s_o^\theta(P,q)$ under vertical integration. The latter term is a weighted average of the rental revenue and revenue from consumers purchasing directly from the network owner on the retail market. If the equilibrium retail price is sufficiently high, then the marginal revenue from an additional consumer on the network will be higher under vertical integration even though the marginal increase in demand will be smaller. The size of this effect will depend on the number of competing retail firms, as more competitors will lead to a decrease in $s_o^\theta(P,q)$ and may lead to a lower retail price for the integrated firm as well[10]. If the integrated price increases in n, then σ^* may briefly decrease in n, but since $\lim_{n\to\infty} p_o = r$, σ^* must eventually begin to increase in n.

Price Mitigation

As quality increases, equilibrium prices also rise. Proposition 1 gives that prices will never rise more than quality and therefore demand will increase, but factor 3 still mitigates the demand increase from the rise in quality. This mitigating effect on owner revenue can be separated into a price component and a magnitude component (similar to factors 2 and 1 respectively). With vertical separation the price component is simply r, the lost rental revenue. However if the network owner is participating in the retail market then the price increase will also

10. Increasing n increases competition, which reduces the markup o charges over its opportunity cost, but it also decreases $s_\theta^o(P,q)$, which causes an increase in price. The effect of an increase in n depends on the relative strength of these two effects.

drive some consumers toward the network owner as well as encouraging them to remain with the outside option, and this will ameliorate the loss in revenue[11]. If p_o is sufficiently high in equilibrium the additional revenue from consumers switching to the network owner can outweigh the loss from consumers remaining with the outside option, meaning that factor 3 will *increase* investment under vertical integration in that circumstance. The magnitude component is simply the size of the mitigation effect, i.e. the number of consumers who choose not to shift to each competitor firm as a result of their price increase. The relative size of this price increase is unclear as the magnitude of demand sensitivity will be larger with vertical separation, but the presence of the cost-advantaged network owner on the retail market may result in a smaller price change.

Horizontal Differentiation

$var(\epsilon_i)$ represents the degree of horizontal differentiation between firms. As the variance increases, the random component of utility becomes more important in consumer decision making (extremely high and low values become more likely) and so consumers become less price sensitive. Reduced price sensitivity diminishes the effect of factor 1 and the magnitude component of factor 3. It alsos increase the equilibrium p_o, which increases the effect of factor 2 and the pushes the price component of 3 toward encouraging investment in quality rather than discouraging it. Thus for highly differentiated firms quality investment is be higher under vertical integration. If the firms are not highly differentiated then the magnitude components take on more importance. In this case the vertically separated regime has an advantage due to the greater impact of quality on demand in that equilibrium. The presence of factor 3 creates some uncertainty about the dominance of the vertically separate equilibrium at low variance as the low retail price of the integrated firm limits the price response of the competitor firms, but the price increase of the integrated firm also encourages the competitor firms to raise their prices more. I have not been able to find general conditions for dominance of the separated equilibrium, but it is generally dominant for low variance under logit demand.

In the limit as the variance goes to 0, competition approaches Bertrand, meaning that prices do not react at all to changes in quality, but in this case there is no incentive to invest in either regime unless $r > u(0)$. If baseline quality is that low, then the network owner either sets $u(q) = r = p_j$ in order to ensure participation of consumers in the market[12], or if it is vertically integrated it may set $p_o = u(q) < r$. This latter case can occur because the integrated owner will set a price equal to utility from quality when $u(q) < r$, and the marginal revenue from increasing quality (and therefore price) is not necessarily sufficient to cause the owner to reach a corner solution at r.

11. It is possible for p_o to rise more than p_j as a result of an increase in q. In this case the interpretation is slightly different but the effect remains. See Appendix A.4 for a detailed discussion.

12. If $\alpha(q) > r$ at this quality, then the owner will instead set $q = 0$ and settle for 0 profit, because the cost of ensuring participation is greater than the revenue.

Example Using Logit Demand

If the ϵ draws follow a Gumbel distribution, then the demand will conform to the standard logit model. Denote the scale parameter of the Gumbel distribution by v, so the variance of the distribution will be $\frac{\pi^2}{6}v^2$ and from standard textbook results (Train 2009), demand for firm i is

$$d_i(P,q) = \frac{\exp(\frac{-p_i}{v})}{\exp(\frac{-u(q)}{v}) + \sum_{k\in\{1,2,\ldots,n,o\}} \exp(\frac{-p_k}{v})}$$

From symmetry, this means that with vertical integration demand is

$$d_i(P,q) = \frac{\exp(\frac{-p_i}{v})}{\exp(\frac{-u(q)}{v}) + n\exp(\frac{-p_j}{v}) + \exp(\frac{-p_o}{v})}$$

and in the separated regime

$$d_i(P,q) = \frac{\exp(\frac{-p_i}{v})}{\exp(\frac{-u(q)}{v}) + (n+1)\exp(\frac{-p_j}{v})} \tag{8}$$

The price at the un-integrated retail firm must then solve[13]

$$p_j = r + v * \frac{\exp(\frac{-u(q)}{v}) + n\exp(\frac{-p_j}{v}) + \exp(\frac{-p_o}{v})}{\exp(\frac{-u(q)}{v}) + (n-1)\exp(\frac{-p_j}{v}) + \exp(\frac{-p_o}{v})} \tag{9}$$

The equilibrium price is not necessarily monotonic in v, but as v goes to infinity the quotient in the second term will converge to $\frac{n+2}{n_1}$ while v is unbounded, so the price must eventually become a strictly increasing and unbounded function of v.

If the network owner is participating in the retail market, p_o solves

$$\begin{aligned} p_o =& r * \frac{n\exp(\frac{-p_j}{v})}{\exp(\frac{-u(q)}{v}) + n\exp(\frac{-p_j}{v})} + v * \frac{\exp(\frac{-u(q)}{v}) + n\exp(\frac{-p_j}{v}) + \exp(\frac{-p_o}{v})}{\exp(\frac{-u(q)}{v}) + n\exp(\frac{-p_j}{v})} \\ =& \frac{v\exp(\frac{-u(q)}{v}) + (v+r)n\exp(\frac{-p_j}{v}) + v\exp(\frac{-p_o}{v})}{\exp(\frac{-u(q)}{v}) + n\exp(\frac{-p_j}{v})} \end{aligned}$$

Again, the price increases unboundedly (although not necessarily monotonically for small v) in v, but it is not necessarily increasing in p_j. For small n and r large relative to v, the decreased appeal of the unintegrated firms to the marginal consumers from an increase in price reduces the opportunity cost of the network owner more than it increases the markup, resulting in a net decrease in price. Finally, the opportunity cost term in the owner's price equation does not depend on p_o. This comes from the the well known independence of irrelevant alternatives property in logit demand models, and implies that assumption 3 issatisfied for all quality levels, meaning that we do not need to place restrictions on $u(0)$[14].

13. To save notation I only show the vertically integrated case. For the vertically separated case we can just set $p_o = p_j$

14. However I do assume that $u(0) > r$ for two reasons: 1. The limiting behavior of prices as $v \to 0$ can be extremely strange if $u(q) < r$ and 2. Since u(q) is endogenous, this assumption avoids boundary conditions where the inequality switches.

If v approaches 0 then all prices will converge to r f(or $u(q) > r$). However, the terms in p_o which are multiplied by v approach 0 faster than the one which is not, so for small v

$$p_o \approx r * \frac{n\exp(\frac{-r}{v})}{\exp(\frac{-u(q)}{v}) + n\exp(\frac{-r}{v})} < r$$

The size of the v which is sufficiently small that $p_o < r$ depends on the quality level and n relative to r, but such v will always exist. p_o is non-monotonic in v in this region, as the right hand side of the equation above will eventually approach r as v becomes sufficiently small.

Fix q and let $\frac{\partial x}{\partial \mathbf{p_j}}$ denote the derivative of x when all competitor firms raise their prices symmetrically[15], similarly define $\frac{\partial \mathbf{p_j}}{\partial x}$ as the symmetric change in competitor prices as a result of a change in x. Quality investment is determined by eq. (17) and eq. (18) in appendix A.4, which can be reproduced in this setting as

$$\frac{\alpha'(q)}{u'(q)} = \left(\frac{rn\exp(\frac{-p_j}{v}) + p_o\exp(\frac{-p_o}{v})}{n\exp(\frac{-p_j}{v}) + \exp(\frac{-p_o}{v})}\right)\left(-\frac{\partial d_\theta(P,q)}{\partial u(q)}\right)$$
$$+ n\left(\frac{r\exp(\frac{-p_j}{v}) - (p_o - r)\exp(\frac{-p_o}{v})}{n\exp(\frac{-p_j}{v}) + \exp(\frac{-p_o}{v})}\right)\frac{\partial d_j(P,q)}{\partial \mathbf{p_j}}\left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]$$

For the integrated equilibrium and

$$\frac{\alpha'(q)}{u'(q)} = r\left(\frac{-\partial d_\theta(P,q)}{\partial u(q)}\right) + r(n+1)\frac{\exp(\frac{-u(q)}{v})}{n\exp(\frac{-p_j}{v}) + \exp(\frac{-u(q)}{v})}\frac{\partial d_j(P,q)}{\partial \mathbf{p_j}}\frac{\partial \mathbf{p_j}}{\partial u(q)}$$

For the separated equilibrium. Using eq. (8) and eq. (9) these can be simplified to

$$\frac{\alpha'(q)}{u'(q)} = \frac{\frac{1-\left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]}{v}\exp(\frac{-u(q)}{v})rn\exp(\frac{-p_j}{v}) + \frac{p_o\exp(\frac{-u(q)}{v}) + n(p_o - r)\exp(\frac{-p_j}{v})\left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]}{v}\exp(\frac{-p_o}{v})}{\left(\exp(\frac{-u(q)}{v}) + n\exp(\frac{-p_j}{v}) + \exp(\frac{-p_o}{v})\right)^2} \tag{10}$$

and

$$\frac{\alpha'(q)}{u'(q)} = r * \frac{\frac{1-\frac{\partial \mathbf{p_j}}{\partial u(q)}}{v}\exp(\frac{-u(q)}{v})(n+1)\exp(\frac{-p_j}{v})}{\left(\exp(\frac{-u(q)}{v}) + (n+1)\exp(\frac{-p_j}{v})\right)^2} \tag{11}$$

Respectively. The exponential terms in the denominator of eq. (11) represent the change in demand for the outside option as $\Delta_{j\theta}$ changes, and the term $\frac{1-\frac{\partial \mathbf{p_j}}{\partial u(q)}}{v}$ is the variance normalized change in $\Delta_{j\theta}$ as $u(q)$ increases. By proposition 1 $\frac{\partial \mathbf{p_j}}{\partial u(q)} < 1$, so this term will

15. This is equivalent to taking all of the derivatives with regard to individual prices and summing the results, but eliminates the need for summing the effects and/or canceling out cross derivatives. In practice it just means treating all of the competitor prices as a single variable when taking derivatives.

always be positive, but also $\frac{\partial \mathbf{p_j}}{\partial u(q)} > 0$, reflecting the price mitigation effect discussed after proposition 2. The first term in the denominator of eq. (10) has the exact same interpretation, except that the price mitigation effect also reflects the implications of the change in p_o. The second term is the effect of the quality change on revenue at the integrated retailer. The first part of the second term is the direct increase in demand as a direct result of the change in quality, and the second part shows the effect of the un-integrated retailers' price increase on demand at the integrated firm.

As $v \to \infty$, the right hand side of eq. (11) approaches 0, reflecting the fact that for sufficiently differentiated products an increase in quality causes minimal change in demand[16]. But the RHS of eq. (10) approaches a constant due to the presence of p_o so for sufficiently high v the investment incentive must be higher for an integrated network owner than a separated one. On the other hand, for v close to 0 but positive, $\frac{\partial \mathbf{p_j}}{\partial u(q)}$ is smaller under vertical integration, but $\frac{\partial \mathbf{p_j}}{\partial p_o}$ is greater than the price response to quality under vertical separation, and, $\frac{\mathrm{d}p_o}{\mathrm{d}u(q)} > 1$ for v close to 0, so the mitigation effect from the price response is greater under integration than separation[17]. Separating the $(n+1)$ in the numerator of the RHS of eq. (11) into the n firms that are separated in both regimes and the one firm which is potentially integrated, These results give that for low v

$$rn\frac{\frac{1-\frac{\partial \mathbf{p_j^{sep}}}{\partial u(q)}}{v}\exp(\frac{-u(q)}{v})\exp(\frac{-p_j^{sep}}{v})}{\left(\exp(\frac{-u(q)}{v})+(n+1)\exp(\frac{-p_j^{sep}}{v})\right)^2} > rn\frac{\frac{1-\left[\frac{\partial \mathbf{p_j^{int}}}{\partial u(q)}+\frac{\partial \mathbf{p_j^{int}}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]}{v}\exp(\frac{-u(q)}{v})\exp(\frac{-p_j^{int}}{v})}{\left(\exp(\frac{-u(q)}{v})+n\exp(\frac{-p_j^{int}}{v})+\exp(\frac{-p_o}{v})\right)^2}$$

Therefore the investment incentive stemming from these n firms is stronger under vertical separation in an undifferentiated market. The question is whether the investment incentive from the integrated firm is greater and if this counteracts the difference in the other n firms. Since v is low, $p_o < r$, and $\frac{\exp(\frac{-u(q)}{v})\exp(\frac{-p_o}{v})}{\left(\exp(\frac{-u(q)}{v})+n\exp(\frac{-p_j}{v})+\exp(\frac{-p_o}{v})\right)^2}$ is increasing in both p_o and p_j so

$$r\frac{\frac{1}{v}\exp(\frac{-u(q)}{v})\exp(\frac{-p_j}{v})}{\left(\exp(\frac{-u(q)}{v})+(n+1)\exp(\frac{-p_j}{v})\right)^2} > p_o\frac{\frac{1}{v}\exp(\frac{-u(q)}{v})\exp(\frac{-p_o}{v})}{\left(\exp(\frac{-u(q)}{v})+n\exp(\frac{-p_j}{v})+\exp(\frac{-p_o}{v})\right)^2}$$

The investment incentive stemming from consumers coming on to the integrated firm from the outside option is lower than that from the separated version of the same firm. The only remaining comparison is to evaluate

$$-r\frac{\frac{\frac{\partial \mathbf{p_j}}{\partial u(q)}}{v}\exp(\frac{-u(q)}{v})\exp(\frac{-p_j}{v})}{\left(\exp(\frac{-u(q)}{v})+(n+1)\exp(\frac{-p_j}{v})\right)^2} \lesseqgtr n(p_o-r)\frac{\frac{\left[\frac{\partial \mathbf{p_j}}{\partial u(q)}+\frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]}{v}\exp(\frac{-p_j}{v})\exp(\frac{-p_o}{v})}{\left(\exp(\frac{-u(q)}{v})+n\exp(\frac{-p_j}{v})+\exp(\frac{-p_o}{v})\right)^2}$$

16. This also shows one of the drawbacks of assuming that the ϵ draws are i.i.d. across all options, as a situation where products are highly differentiated intuitively suggests that firms are monopolists relative to each other, but does not intuitively imply that consumers who choose the outside option are entirely uninterested in every firm. However allowing the outside option to have its own distribution would make the model significantly more complex and would add little intuition.

17. Derivations for the statements in this section are straightforward but tedious and long. They are available upon request.

$p_o - r < 0$ but for $u(q) > r$, $p_o - r > -\frac{r}{n}$, so the revenue loss from consumers staying with the integrated firm as a result of competitors increasing prices is less than the loss from consumers staying with the outside option. Combined with results on the magnitude of changes this means that

$$\frac{n(p_o - r)\exp(\frac{-p_o}{v})}{\left(\exp(\frac{-u(q)}{v}) + n\exp(\frac{-p_j}{v}) + \exp(\frac{-p_o}{v})\right)^2} > \frac{-r\exp(\frac{-p_j}{v})}{\left(\exp(\frac{-u(q)}{v}) + (n+1)\exp(\frac{-p_j}{v})\right)^2}$$

But it is also the case that for $u(q) > r$ and small v

$$\frac{\frac{\partial \mathbf{p_j}}{\partial u(q)}}{v}\exp(\frac{-u(q)}{v}) < \frac{\left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]}{v}\exp(\frac{-p_j}{v})$$

but for large n any differences at this one firm will be minimal compared to the effects at the other firms so the separated firm will have more investment. For low n the results are less clear, but numerical simulations seem to suggest that the separated firm still has a stronger investment incentive. The results of this example are summarized in table 1.

	Low variance	High variance
High n	Separated firm has more investment.	Integrated firm has more investment
Low n	Unknown, but separated firm probably invests more	Integrated firm has more investment

Table 1: Quality investment comparison for various parameter sets.

The Regulator

The regulator's objective is to maximize social welfare, and it chooses a market structure and rental price in order to achieve this goal. Adding together consumer utility and firm profits, the regulator's objective function is

$$\begin{aligned} U =&(1 - d_\theta(P,q))u(q) + \sum_{i=j,o}\int_{-\infty}^{\infty}\epsilon f(\epsilon)F(\epsilon + \Delta_{i\theta})\prod_{k\neq i}F(\epsilon + \Delta_{ik})d\epsilon \\ &+ \int_{-\infty}^{\infty}\epsilon f(\epsilon)\prod_{k}F(\epsilon - \Delta_{k\theta})d\epsilon - \alpha(q) \end{aligned} \tag{12}$$

Prices in the retail and wholesale markets are transfers between agents within the market and therefore cancel out of the welfare function except in their effect on agents' decisions.

Welfare is therefore the sum of utility from consumers participating in the market, the random utility consumers receive from their chosen option, and the cost of providing quality.

When the regulator increases r, the direct effect is to increase the effective marginal costs of all firms as is shown by the first terms of eq. (5), eq. (6), and eq. (7). Indirectly, it will affect the network owner's investment decision, and may reduce the network owner's effective marginal cost of production under vertical integration if the increase in p_j sufficiently reduces $s_j^o(P, q)$[18]. The direct effects are all undesirable from the regulator's perspective as increased prices reduce participation, which in turn reduces the number of consumers receiving the benefits of the network. Therefore the effects of r on investment are of paramount importance to the regulator's decision.

Vertically Separated Equilibrium

Lemma 3. *Under vertical separation, investment is increasing in r at $r = 0$*

Proof. For $r \leq 0$ the network owner receives no benefit from investment and will always choose $q = 0$. If $r > 0$ then the Inada conditions on $\alpha(\cdot)$ and $u(\cdot)$ will ensure that the LHS of Equation (17) will be arbitrarily close to 0 for q arbitrarily close to 0. Since the right hand side will be positive, this implies that investment will increase. ■

Lemma 3 says that a regulator who chooses a vertically separated regime will always set a positive rental price. Investment may or may not be monotonic in r, This in turn lets us characterize the choice of r under vertical separation.

Proposition 3. *If $u(\cdot)$ is sufficiently concave and/or $\alpha(\cdot)$ is sufficiently convex, an optimal choice under vertical separation r^*_{sep} exists. If $arg\max_r q$ exists, then $r^*_{sep} < arg\max_r q$. In all cases the network owner will set q below the socially optimal level.*

Proof. See appendix A.5 ■

The concavity requirement ensures that even if investment is monotonically increasing in r, there will be diminishing social returns to quality investment and increasing social costs. Therefore the direct price increases from increasing r become more important than the indirect effect of increasing investment as r increases. In the case where investment is not monotonic , the marginal quality investment as a result of increasing r goes to 0, while the direct welfare losses from increasing prices do not, so the net marginal social benefit of increasing r is always be below the gross benefit from the increased investment, and the socially optimal rental price must be below the price which maximizes investment[19]. The network owner under-invests because the marginal revenue it receives from investing is less than even the total marginal revenue accrued by the firms in the market, let alone the utility increase from the inframarginal consumers. It therefore does not take these positive externalities into account.

18. Note that a sufficiently high r could cause prices to violate assumption 3, but I am implicitly assuming that either r is below this level or that $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)}$ is constant.

19. If $arg\max_r q$ is not a singleton, then $r^*_{sep} < min(arg\max_r q)$

Vertically Integrated Equilibrium

The effects of r on equilibrium in the vertically integrated regime are broadly similar to those under vertical separation, but the presence of the vertically integrated firm implies heterogeneous firm reactions to the increase in rental price. The direct effect on the un-integrated retail firms is to raise prices as before, but the network owner's retail price can be increasing or decreasing in r. Holding q constant in order to evaluate only the direct effects of r, consider eq. (7). The final term on the RHS, $-\left(\frac{\partial \ln(d_o(P,q))}{\partial p_o}\right)^{-1}$ increases for every value of p_o since the competitors are raising their prices. However, $s_j^o(P,q)$ is decreasing in p_j from lemma 2 so the first term, $nrs_j^o(P,q)$, increases for any given value of p_o only if $s_j^o(P,q) \in o(r^x)$ where $x > -1$[20]. If $s_j^o(P,q) = o(r^{-1})$, then p_o may still increase in r if $-\left(\frac{\partial \ln(d_o(P,q))}{\partial p_o}\right)^{-1}$ increases sufficiently quickly, but otherwise it is non-monotonic in r.

Lemma 4. *Holding q constant, p_o is not a decreasing function of r. $\frac{\partial p_o}{\partial r} > 0$ when $r = 0$.*

Proof. $s_j^o(P,q) \in (0,1]$[21] and $-\left(\frac{\partial \ln(d_o(P,q))}{\partial p_o}\right)^{-1}$ are continuous functions of prices, and p_j is continuous in r from continuity of $-\left(\frac{\partial \ln(d_j(P,q))}{\partial p_j}\right)^{-1}$ and r. Since $0 \leq s_j^o(P,q) \leq 1$, $\lim_{r\to 0} rs_j^o(P,q) = 0$. But then $\frac{\partial p_o}{\partial r} > 0$ at $r = 0$ since $-\left(\frac{\partial \ln(d_o(P,q))}{\partial p_o}\right)^{-1}$ increases with r and $rs_j^o(P,q) > 0$ for $r > 0$. ■

It is noteworthy that $arg\max_r q$ may be 0 under vertical separation[22]. To see this, consider that q^{int} is determined by eq. (18) in appendix A.4 and increases if the RHS of that equation is increasing for every value of q. Fixing an arbitrary value of q, an increase in r will increase p_j relative to p_o and increases both relative to $u(q)$ (since p_o is increasing at $r = 0$). at $r = 0$, $(rns_j^\theta(P,q) + p_o s_o^\theta(P,q))$ must be increasing since $rns_j^\theta(P,q)$ is increasing since it is 0 for $r = 0$ and positive everywhere else, and it is relatively easy to show that $s_o^\theta(P,q)$ is increasing as p_o increases relative to p_j. We have established that $\left(-\frac{\partial d_\theta(P,q)}{\partial q}\right)$ increases as demand for the outside option increases, which must happen given that prices are increasing and q is fixed. From proposition 1 demand for the competitor firms is increasing in q, which means that $s_j^\theta(P,q)\left(-\frac{\partial d_\theta(P,q)}{\partial q}\right) + s_\theta^j(P,q)\frac{\partial d_j(P,q)}{\partial p_j}\frac{\partial p_j}{\partial q} > 0$, and the change in $s_j^\theta(P,q)\left(-\frac{\partial d_\theta(P,q)}{\partial q}\right) + s_\theta^j(P,q)\frac{\partial d_j(P,q)}{\partial p_j}\frac{\partial p_j}{\partial q}$ with regard to r will not matter since it will be multiplied by $r = 0$. This leaves only $-np_o s_o^j(P,q)\frac{\partial d_j(P,q)}{\partial p_j}\frac{\partial p_j}{\partial q}$. This term is positive and p_o is increasing, but the other terms are decreasing and so if the effect is big enough an increase in r could discourage investment.

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20. $o(\cdot)$ here referring to the convergence notation common in the econometrics literature. Given that $s_j^o(P,q)$ is increasing in competitor prices, this condition seems likely to hold.

21. Since ϵ is unbounded above, the share of consumers whose second choice is one of the un-integrated firms will never reach 0.

22. Allowing negative r introduces concerns about existence of the pricing equilibrium, but it is possible that if equilibrium exists a negative rental fee might be optimal

Proposition 4. *For $u(\cdot)$ sufficiently concave and/or $\alpha(\cdot)$ sufficiently convex, and $s_j^o(P,q) = o(r^x)$ where $x > -1$ an optimal r^{int} exists under vertical integration. The network owner may over-invest or under-invest in network quality compared to the social optimum. If $arg\max_r q$ exists, then $r^{int} < arg\max_r q$.*

Proof. See Appendix A.6

■

Requiring $s_j^o(P,q) = o(r^x)$ with $x > -1$, ensures that a solution to the regulator's problem exists. If p_o is allowed to decrease in response to an increase in r then social welfare may strictly increase with the rental price.

There are four sources of misalignment between the network owner's investment decision and the regulator's:

1. If $p_o > r$, network owner benefits from consumers switching to its retail firm away from the competitor firms (or not switching away after a price increase), where this change makes no difference from a welfare perspective.

2. The network owner only partially accounts for the mitigating effect of the un-integrated retail firm's price increases in response to quality increasing.

The network owner does not account for:

3. The welfare mitigating effect of its own price increase

4. The utility gain from consumers already participating on the market

Over-investment can occur if effects 1,2, and 3 are relatively strong compared to effect 4. This can occur if $var(\epsilon_i)$ is large enough so that the profit implications of 1 are impactful, but not so large that the mitigation welfare losses from 2 and 3 are negligible. In this case the regulator may wish to choose the vertically separated equilibrium despite higher prices and lower investment at any given retail price.

3 Future Work

There are two avenues of extension I am considering to complete this paper. The first is to solve for equilibrium when the network owner is setting the rental price. The result that the vertically separated owner can have a stronger investment incentive is unusual in the literature, and it is unclear whether this comes from the increased level of competition compared to other papers (most vertical integration models use serial monopolies) or the regulated rental price.

The other avenue is to continue the logit demand example to to the regulator's section. I am not sure how much additional insight this will add since the main contribution of this paper is in the comparison of investment incentives under the different regimes. I welcome comments on both avenues of extension.

An interesting question that is not considered in this paper would be to consider the effects of credit limitations on discrete quality investments. Since network investment requires a large up front fee, one could easily imagine access regulation allowing a small firm to grow until it has the ability to invest in its own infrastructure. Whereas without access it would not have the ability to operate at that smaller scale while accruing the funds necessary for the network investment. There has been some work in this area, Hori and Mizuno (2006) look at the effect of access pricing on incentives to engage in "replacement" investment. That is replacing a preexisting network with an upgraded, higher quality version (the example of choice being replacement of a copper internet network with fiber optic cable). However they only consider an equilibrium where the market is large enough that a competitor will eventually enter regardless of access pricing. The network owner invests in an upgraded network to induce the competitor to delay the creation of its own network. Access to the network increases the appeal of this delay to the competitor, which gives the network owner more incentive to invest. Bourreau, Cambini, and Dogan (2012) formulate a model that is somewhat similar to this idea, though in their formulation the "new generation network" is not subject to access regulation. Access to the "old generation network" increases the opportunity cost to the competitor of investing in their own network, and so reduces the incentive of the entrant to invest in new technology. If we were to add credit limitations into the mix, a vertically integrated network owner would have more tools with which to reduce the profits of potential competitors in the wholesaler space, and therefore might set prices artificially low as a kind of limit pricing strategy.

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A Proofs

A.1 Proof of Lemma 1

Taking the derivatives of eq. (2) and eq. (3) we get the following identities

$$\frac{\partial d_j(P,q)}{\partial p_j} = -\int_{-\infty}^{\infty} f(\epsilon)f(\epsilon+\Delta_{j\theta})\prod_{k\neq j}F(\epsilon+\Delta_{jk}) + \sum_{\ell\neq j} f(\epsilon)F(\epsilon+\Delta_{j\theta})f(\epsilon+\Delta_{j\ell})\prod_{k\neq j,\ell}F(\epsilon+\Delta_{jk})d\epsilon \quad (13)$$

$$\frac{\partial d_i(P,q)}{\partial p_j} = \int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{i\theta})f(\epsilon+\Delta_{ij})\prod_{k\neq i,j}F(\epsilon+\Delta_{ik})d\epsilon \quad (14)$$

$$\frac{\partial d_\theta(P,q)}{\partial p_j} = \int_{-\infty}^{\infty} f(\epsilon)f(\epsilon-\Delta_{j\theta})\prod_{k\neq j}F(\epsilon-\Delta_{k\theta})d\epsilon \quad (15)$$

Using the substitution rule for eq. (14), letting $\epsilon^* = \epsilon + \Delta_{ij}$

$$\frac{\partial d_i(P,q)}{\partial p_j} = \int_{-\infty}^{\infty} f(\epsilon^*+\Delta_{ji})F(\epsilon^*+\Delta_{j\theta})f(\epsilon^*)\prod_{k\neq i,j}F(\epsilon^*+\Delta_{jk})d\epsilon^*$$

Note that the unbounded integral means we do not need to consider the bounds of integration. I consider the case with a finite lower bound on the support of $f(\cdot)$ in Appendix B. Repeating this exercise for all $i \neq j$, we can then see that

$$\sum_{i\neq j}\frac{\partial d_i(P,q)}{\partial p_j} = \int_{-\infty}^{\infty} \sum_{i\neq j} f(\epsilon^*+\Delta_{ji})F(\epsilon^*+\Delta_{j\theta})f(\epsilon^*)\prod_{k\neq i,j}F(\epsilon^*+\Delta_{jk})d\epsilon^*$$

Similarly, letting $\epsilon^{**} = \epsilon + \Delta_{j\theta}$

$$\frac{\partial d_\theta(P,q)}{\partial p_j} = \int_{-\infty}^{\infty} f(\epsilon^{**}+\Delta_{j\theta})f(\epsilon^{**})\prod_{k\neq j}F(\epsilon^{**}+\Delta_{jk})d\epsilon^{**}$$

Substituting these identities into eq. (13)

$$\frac{\partial d_j(P,q)}{\partial p_j} = -\left[\frac{\partial d_\theta(P,q)}{\partial p_j} + \sum_{i\neq j}\frac{\partial d_i(P,q)}{\partial p_j}\right]$$

The result follows. ■

A.2 Proof of Lemma 2

Proof of (i)

Consider the ratio

$$\frac{\frac{\partial d_\theta(P,q)}{\partial p_o}}{\frac{\partial d_j(P,q)}{\partial p_o}} = \frac{s^o_\theta(P,q)}{s^o_j(P,q)} \tag{16}$$

When p_o increases the change in this ratio will be

$$\frac{\frac{\partial^2 d_\theta(P,q)}{\partial^2 p_o}\frac{\partial d_j(P,q)}{\partial p_o} - \frac{\partial d_\theta(P,q)}{\partial p_o}\frac{\partial^2 d_j(P,q)}{\partial^2 p_o}}{\frac{\partial d_j(P,q)}{\partial p_o}^2}$$

So the which means that the ratio will decrease if

$$\frac{\frac{\partial^2 d_\theta(P,q)}{\partial^2 p_o}}{\frac{\partial d_\theta(P,q)}{\partial p_o}} < \frac{\frac{\partial^2 d_j(P,q)}{\partial^2 p_o}}{\frac{\partial d_j(P,q)}{\partial p_o}}$$

Taking the expanded form and separating $F(\epsilon - \Delta_{j\theta})^n$ into $F(\epsilon - \Delta_{j\theta})^{n-1}F(\epsilon - \Delta_{j\theta})$

$$\frac{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon-\Delta_{j\theta})f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^{n-1}d\epsilon}{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon-\Delta_{j\theta})f(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^{n-1}d\epsilon} < \frac{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{j\theta})f'(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon}{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{j\theta})f(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon}$$

Letting $\epsilon^{**} = \epsilon - \Delta_{j\theta}$ and using the substitution rule, eq. (16) becomes

$$\frac{\int_{-\infty}^{\infty} f(\epsilon^{**}+\Delta_{j\theta})F(\epsilon^{**})f'(\epsilon^{**}-\Delta_{oj})F(\epsilon^{**})^{n-1}d\epsilon^{**}}{\int_{-\infty}^{\infty} f(\epsilon^{**}+\Delta_{j\theta})F(\epsilon^{**})f(\epsilon^{**}-\Delta_{oj})F(\epsilon^{**})^{n-1}d\epsilon^{**}} < \frac{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{j\theta})f'(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon}{\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{j\theta})f(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon}$$

Strict log concavity gives that $\frac{f'(\epsilon)}{f(\epsilon)}$ is decreasing in ϵ, so using Theorem 2 in Wijsman (1985), the above inequality holds if $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)}$ is decreasing in ϵ, the two sides are equal if it is constant, and the inequality is reversed if $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)}$ is increasing.

From symmetry, $n\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}} = \sum_j \frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}}$, and from Lemma 1 $\sum_j \frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}} = \sum_{k\neq\theta} s_k^o(P,q) = 1 - s_\theta^o(P,q)$. If eq. (16) holds, then θ's relative share of the consumers leaving o will be decreasing, so $n\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}} = 1 - s_\theta^o(P,q)$ will increase. Similarly, if eq. (16) is replaced with equality or the inequality is reversed, then $n\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}}$ is constant or decreasing in p_o respectively. Since $f(\cdot)$ is log concave, $\frac{f(\cdot)}{F(\cdot)}$ is a non-increasing positive function, and therefore approaches some lower limit as $\epsilon \to \infty$ if it is not constant. But then $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)}$ is either be constant or monotonically approaches 1 as ϵ increases. If $u(q) - p_j > 0$ ($\Delta_{j\theta}$) positive) then $\frac{f(\epsilon+\Delta_{j\theta})}{F(\epsilon+\Delta_{j\theta})}\frac{F(\epsilon)}{f(\epsilon)} < 1\forall\epsilon$ and is be an increasing function. Similarly, it will be constant or decreasing for $u(q) - p_j = 0$ and $u(q) - p_j < 0$ respectively.

Proof of (ii)

From the above $n\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}} = \sum_{k\neq\theta} s_k^o(P,q) < 1$, the result follows immediately.

Proof of (iii)

As $n \to \infty$, $s_\theta \to 0$, so $\frac{\frac{\partial d_j}{\partial p_o}}{\sum_j \frac{\partial d_j}{\partial p_o} + \frac{\partial d_\theta}{\partial p_o}}$ will approach $\frac{1}{n}$, but then $nr\frac{\frac{\partial d_j}{\partial p_o}}{-\frac{\partial d_o}{\partial p_o}}$ converges to $n\frac{1}{n}r = r$

Proof of (iv)

Any fraction of the form $\frac{x}{x+y}$ will increase (decrease) if x increases (decreases) relative to y.

$nrs_j^o(P,q)$ has the form $r\frac{n\frac{\partial d_j}{\partial p_o}}{n\frac{\partial d_j}{\partial p_o} + \frac{\partial d_\theta}{\partial p_o}}$, so using the fact that from eq. (14), eq. (15) and symmetry of the competitor firms $ns_j^o(P,q)$ increases (decreases) if $\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{j\theta})f(\epsilon-\Delta_{oj})F(\epsilon)^n d\epsilon$ increases (decreases) relative to $\int_{-\infty}^{\infty} f(\epsilon)f(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon$. Taking the derivative of each with regard to $u(q)$, this means that $ns_j^o(P,q)$ increases if

$$\begin{aligned}\int_{-\infty}^{\infty} f(\epsilon)f(\epsilon+\Delta_{j\theta})f(\epsilon-\Delta_{oj})F(\epsilon)^n d\epsilon > -\Big(&\int_{-\infty}^{\infty} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n \\ &+ nf(\epsilon)f(\epsilon-\Delta_{o\theta})f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^{n-1}d\epsilon\Big) \\ = -\Big(&\int_{-\infty}^{\infty} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon \\ &+ \int_{-\infty}^{\infty} nf(\epsilon)f(\epsilon-\Delta_{o\theta})f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^{n-1}d\epsilon\Big)\end{aligned}$$

The LHS of the above inequality is positive. If $f(\cdot)$ is symmetric, then $f'(\cdot)$ is positive until some critical point x^*, after which it is negative, so we can separate $\int_{-\infty}^{\infty} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-$

$\Delta_{j\theta})^n d\epsilon$ into $\int_{-\infty}^{x^*+\Delta_{o\theta}} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon$ and $\int_{x^*+\Delta_{o\theta}}^{\infty} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon$. From symmetry $\int_{-\infty}^{x^*+\Delta_{o\theta}} f'(\epsilon-\Delta_{o\theta})d\epsilon = -\int_{x^*+\Delta_{o\theta}}^{\infty} f'(\epsilon-\Delta_{o\theta})d\epsilon$, and $F(x^*) = \frac{1}{2}$. Since $\Delta_{o\theta} > \Delta_{j\theta}$ this means $\int_{-\infty}^{x^*+\Delta_{o\theta}} f(\epsilon)F(\epsilon-\Delta_{j\theta})^n d\epsilon > \int_{x^*+\Delta_{o\theta}}^{\infty} f(\epsilon)F(\epsilon-\Delta_{j\theta})^n d\epsilon$, so

$$\left|\int_{-\infty}^{x^*+\Delta_{o\theta}} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon\right| > \left|\int_{x^*+\Delta_{o\theta}}^{\infty} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon\right|$$
$$\implies \int_{-\infty}^{x^*+\Delta_{o\theta}} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon + \int_{x^*+\Delta_{o\theta}}^{\infty} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon > 0$$

So

$$-\left(\int_{-\infty}^{\infty} f(\epsilon)f'(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon + \int_{-\infty}^{\infty} nf(\epsilon)f(\epsilon-\Delta_{o\theta})f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^{n-1} d\epsilon\right) < 0$$

since the second integral consists of non-negative functions, so $s_j^o(P,q)$ increases in q. By a nearly identical argument $\int_{-\infty}^{\infty} f(\epsilon)F(\epsilon+\Delta_{j\theta})f(\epsilon-\Delta_{oj})F(\epsilon)^n d\epsilon$ decreases in p_j while $\int_{-\infty}^{\infty} f(\epsilon)f(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon$ increases, so $s_j^o(P,q)$ will decreases in p_j
■

A.3 Proof of Proposition 1

Existence

In the equilibrium with a vertically separate network owner, the pricing subgame is exactly equivalent to the supermodular pricing game in lemma 1 of Quint (2014), and from that lemma this subgame then has a unique equilibrium for every level of q and r. From symmetry of the firms in this equilibrium, this equilibrium must be symmetric.

For the competitor firms in the vertically separated equilibrium, log concavity of demand is sufficient to ensure sufficiency of the first order condition. For the network owner, consider

$$\frac{\partial V_o}{\partial p_o} = d_o(P,q) + p_o\frac{\partial d_o(P,q)}{\partial p_o} + r\sum_j \frac{\partial d_j(P,q)}{\partial p_o}$$

After imposing symmetry

$$\frac{\partial V_o}{\partial p_o} = d_o(P,q) + p_o\frac{\partial d_o(P,q)}{\partial p_o} + nr\frac{\partial d_j(P,q)}{\partial p_o}$$

Using lemma 1 and combining terms

$$\begin{aligned}\frac{\partial V_o}{\partial p_o} =& d_o(P,q) + p_o \frac{\partial d_o(P,q)}{\partial p_o} - nrs_j^o(P,q)\frac{\partial d_o(P,q)}{\partial p_o} \\ =& d_o(P,q) + \left(p_o - nrs_j^o(P,q)\right)\frac{\partial d_o(P,q)}{\partial p_o} \\ =& d_o(P,q)\left[1 + \left(p_o - nrs_j^o(P,q)\right)\frac{\frac{\partial d_o(P,q)}{\partial p_o}}{d_o(P,q)}\right]\end{aligned}$$

Since $\frac{\partial d_o(P,q)}{\partial p_o} < 0$, This derivative is positive at $p_o = 0$ and from lemma 2 and assumption 3 $s_j^o(P,q)$ is non-increasing in p_o , so $p_o - nrs_j^o(P,q)$ is increasing in p_o. Combined with log concavity of $d_o(P,q)$, this means that $\left[1 + \left(p_o - nrs_j^o(P,q)\right)\frac{\frac{\partial d_o(P,q)}{\partial p_o}}{d_o(P,q)}\right]$ is strictly decreasing in p_o. Since demand in the random utility framework is strictly positive, this means that $\frac{\partial V_o}{\partial p_o}$ is positive at $p_o = 0$, and can go from positive to negative at most one time. o's profit will be quasi-concave in its own price, and the only possible corner solution ($p_o = 0$) is ruled out by the positive derivative at that point, the first order condition is sufficient for maximization.

The remainder of this proof borrows heavily from the proof of Theorem 1 in Quint (2014). It is easy to verify that eq. (5) is equivalent to $p_j = r - \left(\frac{\partial \ln(d_j(P,q)}{\partial p_j}\right)^{-1}$. From theorem 1 in Quint (2014) log demand has increasing differences in the prices of other firms, so the optimal price of the un-integrated firms is increasing in the prices of the other firms. If we take the limit as all other firms' prices go to infinity the effective price of the outside option ($u(q)$) will still be finite, so this limit will still be finite, meaning that the un-integrated firms' best response functions are bounded above. Looking at V_j, any price below r will be dominated. Similarly, $p_o = nrs_j^o(P,q) - \left(\frac{\partial \ln(d_o(P,q)}{\partial p_o}\right)^{-1}$ is also be bounded above, and from lemma 2 and lemma 1 $0 \leq nrs_j^o(P,q) < r$, so prices below 0 will be dominated, and the best responses for the network owner will also be bounded above. Since prices below 0 and above some upper bound are dominated, we can restrict prices to a closed interval between 0 and some arbitrarily large upper bound (this upper bound may depend on q and r) without any effect on the best responses. The strategy spaces are therefore compact and convex, and the payoff functions are continuous in all prices and quasi-concave in own price, so we can apply theorem 1.2 from Fudenberg and Tirole (1991), meaning that the pricing subgame has at least one pure strategy equilibrium.

Uniqueness

Now suppose we have two equilibria with price vectors P and P', then letting $i \in arg\max_k |p_k^{'} - p_k|$ and without loss of generality suppose $p_k^{'} > p_k$, set $\Delta_k^{'} = p_k^{'} - p_k$. If we add $\Delta_k^{'}$ to all prices and $u(q)$, then the relative appeal of all options will be unchanged and so if we let $P_{-i} + \Delta_k^{'}$ represent adding this difference to all prices in the price vector P_{-i} of prices not including i's price, and at a slight abuse of notation[23] let $q + \Delta_k^{'}$ represent adding $\Delta_k^{'}$ to

23. this will be the only such abuse in this paper, in all other cases the reader should interpret $q + x$ as adding x to q directly.

$u(q)$, then

$$\frac{\partial \ln(d_i(p_i + \Delta_k^{'}, P_{-i}, q)}{\partial p_i} = \frac{\partial \ln(d_i(p_i + \Delta_k^{'}, P_{-i} + \Delta_k^{'}, q + \Delta_k^{'})}{\partial p_i} > \frac{\partial \ln(d_i(p_i + \Delta_k^{'}, P_{-i}^{'}, q)}{\partial p_i}$$

The inequality comes from the fact that q is constant in both equilibria, and since $\Delta_k^{'}$ is the maximal difference between prices in the two equilibria, for all $i' \neq i$, $p_i^{'} \leq p_i + \Delta_k^{'}$, and increasing differences will give the inequality. This means that i cannot be one of the un-integrated firms since the results above would require the LHS of $p_j = r - \left(\frac{\partial \ln(d_j(P,q)}{\partial p_j}\right)^{-1}$ to increase while the RHS decreases. If $i = o$, then in order to have $p_o^{'} > p_o$, we need $s_j^o(p_o^{'}, P_{-o}^{'}, q)) > s_j^o(p_o, P_{-o}, q))$ and this increase must be sufficient to make up for the decrease in $-\left(\frac{\partial \ln(d_j(P,o)}{\partial p_o}\right)^{-1}$. But from assumption 3 and lemma 2 $s_j^o(p_o, P_{-o}, q))$ is decreasing in p_j, and p_o. Since q is constant and $i = o$ implies that p_o has increased, we must have $p_j^{'} < p_j$, but the relative prices of the competitor firms will be constant due to symmetry of the competitor firms (and even if they were not, we could simply pick the one with the most sizable price decrease between the two equilibria), q is constant and p_o has risen, so $\frac{\partial \ln(d_j(P',q))}{\partial p_j} > \frac{\partial \ln(d_j(P,q))}{\partial p_j}$, which implies that $-\left(\frac{\partial \ln(d_j(P',q)}{\partial p_j}\right)^{-1} > -\left(\frac{\partial \ln(d_j(P',q)}{\partial p_j}\right)^{-1}$, while $p_j^{'} < p_j$, so eq. (5) cannot hold in both equilibria, giving a contradiction.

Lowest price for network owner

Now suppose that the network owner has the highest price in equilibrium. Then

$$p_j = r - \left(\frac{\partial \ln(d_j(P,q)}{\partial p_j}\right)^{-1} < p_o = p_o = nrs_j^o(P,q)) - \left(\frac{\partial \ln(d_o(P,q)}{\partial p_o}\right)^{-1}$$

But consumers treat the firms symmetrically aside from price, so the only difference between $\frac{\partial \ln(d_j(P,q)}{\partial p_j}$ and $\frac{\partial \ln(d_o(P,q))}{\partial p_o}$ will be that the latter function has a higher own price and lower competitor price vector. But then from log concavity and increasing differences it must be the case that $\frac{\partial \ln(d_o(P,q))}{\partial p_o} < \frac{\partial \ln(d_j(P,q))}{\partial p_j}$. If $\frac{\partial \ln(d_o(P,q))}{\partial p_o} < \frac{\partial \ln(d_j(P,q))}{\partial p_j}$ then $-\left(\frac{\partial \ln(d_o(P,q)}{\partial p_o}\right)^{-1} < -\left(\frac{\partial \ln(d_j(P,q)}{\partial p_j}\right)^{-1}$ and from lemma 1 $nrs_j^o(P,q) < r$, so we cannot have all first order conditions hold if $p_o > p_j$. Since the competitor firms are symmetric, p_o will therefore be the lowest price.

Lower prices with integrated firm

Now suppose that some prices in the equilibrium with the integrated firm are higher than in the equilibrium with vertical separation. Let the supercript "sep" denote the separated equilibrium and "int" the integrated equilibrium. $j^* \in arg\max_i p_i^{int}$ where $\{p_i\}$ is the set of prices in the integrated firm equilibrium. Then j^* must be one of the competitor firms since we have shown that p_o cannot be the maximal price, by the same logic as in the uniqueness proof, the price increase for j^* will be equivalent to a price decrease for all of the competitor firms but then $p_{j^*}^{int} > p_{j^*}^{sep}$ and $-\left(\frac{\partial \ln(d_{j^*}(P^{int},q)}{\partial p_{j^*}^{int}}\right)^{-1} < -\left(\frac{\partial \ln(d_{j^*}(P^{sep},q)}{\partial p_{j^*}^{sep}}\right)^{-1}$ so eq. (5) cannot hold in both equilibria.

The relationship between prices and quality

Since $u(q)$ is the effective price of the outside option, $\frac{\partial \ln(d_i(P,q)}{\partial p_i}$ will be increasing in q for all i, but in order for eq. (5) to hold, both sides of the equation must increase so the increase in p_j must be less than the increase in $u(q)$. The comparison for eq. (7) is more complex and $\Delta_{o\theta}$ is not guaranteed to increase in q.

Under the vertically separated equilibrium, an increase in q will result in an increase in prices. Since the equilibrium in that subgame is unique and symmetric for every q and r, the relative appeal of the firms to each other is unchanged, so in order for the first order condition to hold, the relative appeal of the outside option must decrease. Therefore $u(q) - p_j^{sep}$ is increasing in q. Since $p_j^{sep} > p_j^{int} > p_o$, $u(q) - p_j^{sep} < u(q) - p_j^{int} < u(q) - p_o$.

Since prices are increasing in q, and quality investment may be higher with the vertically integrated firm, equilibrium prices may be higher with the integrated firm, but since $u(q) - p_j^{sep}$ is increasing in q and $u(q) - p_j^{int} > u(q) - p_j^{sep}$ for all q, it must be that $u(q') - p_j^{\prime int} > u(q') - p_j^{\prime sep} > u(q) - p_j^{sep}$ for $q' > q$.

■

A.4 Proof of Proposition 2

Using the notation first defined in section 2, $\frac{\partial x}{\partial \mathbf{p_j}}$ denotes the derivative of x when all competitor firms raise their prices symmetrically and $\frac{\partial \mathbf{p_j}}{\partial x}$ the symmetric change in competitor prices as a result of a change in x.

Taking the derivative of network owner profit with regard to q, we get that under vertical separation

$$\begin{aligned}\frac{dV_o}{dq} =& \frac{\partial V_o}{\partial q} + \frac{\partial V_o}{\partial \mathbf{p_j}} \frac{\partial \mathbf{p_j}}{\partial u(q)} u'(q) \\ =& u'(q) r(n+1) \left(\frac{\partial d_j(P,q)}{\partial u(q)} + \frac{\partial d_j(P,q)}{\partial \mathbf{p_j}} \frac{\partial \mathbf{p_j}}{\partial u(q)} \right) - \alpha'(q)\end{aligned}$$

The Inada conditions on $\alpha(\cdot)$ and $u(\cdot)$ are sufficient to ensure that there is no corner solution at $q = 0$, and I assume sufficient convexity and concavity in $\alpha(\cdot)$ and $u(\cdot)$ respectively that the first order condition is sufficient for maximization. This implies that in equilibrium

$$\frac{\alpha'(q)}{u'(q)} = r(n+1) \left(\frac{\partial d_j(P,q)}{\partial u(q)} + \frac{\partial d_j(P,q)}{\partial \mathbf{p_j}} \frac{\partial \mathbf{p_j}}{\partial u(q)} \right)$$

With vertical integration

$$\frac{dV_o}{dq} = \frac{\partial V_o}{\partial q} + \frac{\partial V_o}{\partial p_o} \frac{\mathrm{d}p_o}{\mathrm{d}u(q)} u'(q) + \frac{\partial V_o}{\partial \mathbf{p_j}} \left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o} \frac{\mathrm{d}p_o}{\mathrm{d}u(q)} \right] u'(q)$$

But since p_o is chosen to maximize profit, $\frac{\partial V_o}{\partial p_o} = 0$ from the envelope theorem. Taking the first order condition

$$\begin{aligned}\frac{\alpha'(q)}{u'(q)} =& rn\left(\frac{\partial d_j(P,q)}{\partial u(q)} + \frac{\partial d_j(P,q)}{\partial \mathbf{p_j}}\left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]\right)\\ &+ \frac{\partial d_o(P,q)}{\partial u(q)}p_o + n\frac{\partial d_o(P,q)}{\partial \mathbf{p_j}}\left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]p_o\end{aligned}$$

From result 6 in proposition 1, the demand for network services increases and the change in p_j is smaller than the increase in $u(q)$ even after taking supermodularity of prices into account, so the first term most be positive. The second term is the increased demand at the integrated retailer after a quality increase, and the third term is an upward trend in $d_o(P,q)$ as a result of the increase in p_j[24], and the consumers who stay with the outside option as a result of j increasing its price[25]. Rearranging and using lemma 1, the FOC for vertical separation becomes

$$\frac{\alpha'(q)}{u'(q)} = (rns_j^\theta(P,q) + rs_j^\theta(P,q))\left(-\frac{\partial d_\theta(P,q)}{\partial u(q)}\right) + (n+1)rs_\theta^j(P,q)\frac{\partial d_j(P,q)}{\partial \mathbf{p_j}}\frac{\partial \mathbf{p_j}}{\partial u(q)} \quad (17)$$

and with vertical integration

$$\frac{\alpha'(q)}{u'(q)} = (rns_j^\theta(P,q) + p_o s_o^\theta(P,q))\left(-\frac{\partial d_\theta(P,q)}{\partial u(q)}\right) + n\left(rs_\theta^j(P,q) - s_o^j(P,q)(p_o - r)\right)\frac{\partial d_j(P,q)}{\partial \mathbf{p_j}}\left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right] \quad (18)$$

Fixing a value of q and comparing the RHS of each equation, the first parenthesis is a weighted sum of the revenue the network owner gets from consumers leaving the outside option for each firm. In the vertically separate case, the weights are degenerate and it can be decomposed into $(n+1)rs_j^\theta(P,q) = r$[26], but I leave it in its current form for ease of comparison. If $p_o < r$ then it is always the case that

$$rns_j^\theta(P,q) + p_o s_o^\theta(P,q) < rns_j^\theta(P,q) + rs_j^\theta(P,q)$$

If $p_o > r$ then the inequality is reversed.

From part (iv) of lemma 2, $\frac{\partial^2 d_\theta(P,q)}{\partial p_o \partial q} < 0$, and by a nearly identical argument $\frac{\partial^2 d_\theta(P,q)}{\partial p_j \partial q} < 0$, but then from symmetry of second derivatives $-\frac{\partial d_\theta(P,q)}{\partial q}$ must be larger in the separated equilibrium, so if $p_o < r$ then the revenue from consumers coming on to the network from the outside option is greater under vertical separation than with vertical integration.

For the second term, if $p_o > r$ then

$$(n+1)rs_\theta^j(P,q) > n\left(rs_\theta^j(P,q) - s_o^j(P,q)(p_o - r)\right)$$

24. Note that the total change in $d_o(P,q)$ may be negative if $\frac{\mathrm{d}p_o}{\mathrm{d}u(q)} > \frac{\partial p_j}{\partial u(q)}$, but this reduction in demand is accounted for by the earlier envelope theorem argument

25. Since $u(q) - p_j$ will increase, j will not lose consumers to the outside option, but the increase in demand will not be as large as if it didn't increase its price

26. Without the presence of the network owner in the retail market, symmetry will give $s_j^\theta(P,q) = \frac{1}{n+1}$

If p_o is sufficiently large then the RHS may even be below 0. In that case, the revenue from consumers shifting to the network owner from the competitor firms outweighs the loss in potential revenue from consumers staying with the outside option as prices increase[27]. For smaller values of p_o, the comparison is ambiguous because while

$$(n+1)rs_\theta^j(P,q) > n\left(rs_\theta^j(P,q) - s_o^j(P,q)(p_o - r)\right)$$

the comparison between $\frac{\partial d_j(P,q)}{\partial \mathbf{p_j}}\left[\frac{\partial \mathbf{p_j}}{\partial u(q)} + \frac{\partial \mathbf{p_j}}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right]$ and $\frac{\partial d_j(P,q)}{\partial \mathbf{p_j}}\frac{\partial \mathbf{p_j}}{\partial u(q)}$ at the prices for their respective equilibria is not clear.

As the variance of the ϵ_i increase, consumers will become less price responsive ($\frac{\partial d_i(P,q)}{\partial p_i}$ will decrease for all i). This will have two results:

- $-\frac{\partial d_\theta(P,q)}{\partial q}$ will decrease in magnitude for both equilibria
- p_o will increase

Since $-\frac{\partial d_\theta(P,q)}{\partial q}$ is strictly positive and decreasing, the difference between its values under vertical integration and separation must go to 0 as the $var(\epsilon_i)$ increase. Since p_o is increasing without an upper bound, the effects discussed above (increased marginal revenue and marginal revenue from consumers leaving competitor firms) will dominate the lower demand response from consumers in response to quality increases, so if $var(\epsilon_i)$ is sufficiently high then vertical integration will result in higher investment. For lower values of $var(\epsilon_i)$ the result is ambiguous and depends on the relative strength of the three effects discussed above.

■

A.5 Proof of Proposition 3

Define $\frac{\partial x}{\partial \mathbf{\Delta_{j\theta}}}$ analogously to $\frac{\partial d_i(P,q)}{\partial \mathbf{p_j}}$. Taking the derivative of total welfare with regard to r

$$\begin{aligned}\frac{\partial U}{\partial r} =& \Bigg[\Bigg((1 - d_\theta(P,q)) + \Big[-\frac{\partial d_\theta(P,q)}{\partial \mathbf{\Delta_{j\theta}}}u(q) + \sum_j \int_{-\infty}^{\infty} \epsilon f(\epsilon) f(\epsilon + \Delta_{j\theta}) F(\epsilon)^n d\epsilon \\ &- \int_{-\infty}^{\infty} \epsilon f(\epsilon)(n+1) f(\epsilon - \Delta_{j\theta}) F(\epsilon - \Delta_{j\theta})^n d\epsilon\Big] \frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}\Bigg) u'(q) - \alpha'(q)\Bigg] \frac{\partial q}{\partial r} \\ &- \Big[\frac{\partial d_\theta(P,q)}{\partial \mathbf{p_j}} u(q) + \sum_j \int_{-\infty}^{\infty} \epsilon f(\epsilon) f(\epsilon + \Delta_{j\theta}) F(\epsilon)^n d\epsilon \\ &- \int_{-\infty}^{\infty} \epsilon f(\epsilon)(n+1) f(\epsilon - \Delta_{j\theta}) F(\epsilon - \Delta_{j\theta})^n d\epsilon\Big] \frac{\partial \mathbf{p_j}}{\partial r}\end{aligned}$$

The first half of the derivative is the marginal welfare resulting from an change in q as a result of r. Focusing on that part and expanding $\frac{\partial d_\theta(P,q)}{\partial \mathbf{\Delta_{j\theta}}}$, we get

27. If p_o increases more than p_j in response to a quality increase, then we can interpret this as the revenue from retaining consumers as a result of the increase in p_j.

$$\left[\left((1-d_\theta(P,q))+\Big[\int_{-\infty}^{\infty}(u(q)-\epsilon)f(\epsilon)(n+1)f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon + \sum_j \int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{j\theta})F(\epsilon)^n d\epsilon\Big]\frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}\right)u'(q)-\alpha'(q)\right]\frac{\partial q}{\partial r}$$

From symmetry

$$\sum_j \int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{j\theta})F(\epsilon)^n d\epsilon = (n+1)\int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{j\theta})F(\epsilon)^n$$

Using the substitution rule with $\epsilon^* = \epsilon + \Delta_{j\theta}$

$$(n+1)\int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{j\theta})F(\epsilon)^n = (n+1)\int_{-\infty}^{\infty}(\epsilon^*-\Delta_{j\theta})f(\epsilon^*-\Delta_{j\theta})f(\epsilon^*)F(\epsilon^*-\Delta_{j\theta})^n\epsilon^*$$

Which means

$$\int_{-\infty}^{\infty}(u(q)-\epsilon)f(\epsilon)(n+1)f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon + \sum_j \int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{j\theta})F(\epsilon)^n d\epsilon$$

$$=$$

$$(n+1)\left[\int_{-\infty}^{\infty}(u(q)-\epsilon)f(\epsilon)f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon + \int_{-\infty}^{\infty}(\epsilon^*-\Delta_{j\theta})f(\epsilon^*-\Delta_{j\theta})f(\epsilon^*)F(\epsilon^*-\Delta_{j\theta})^n\epsilon^*\right]$$

$$=$$

$$(n+1)\left[\int_{-\infty}^{\infty}p_j f(\epsilon)f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon\right]$$

Where the second equality comes from combining the integrals and canceling common terms. Plugging this back into the original derivative, we get

$$\begin{aligned}\frac{\partial U}{\partial r} =& \left[\left((1-d_\theta(P,q))+\left[(n+1)p_j\int_{-\infty}^{\infty}f(\epsilon)f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon\right]\frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}\right)u'(q)-\alpha'(q)\right]\frac{\partial q}{\partial r}\\ &-\left[(n+1)p_j\int_{-\infty}^{\infty}f(\epsilon)f(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^n d\epsilon\right]\frac{\partial \mathbf{p_j}}{\partial r}\\ &= \left[\left((1-d_\theta(P,q))-p_j\frac{\partial d_\theta(P,q)}{\partial \mathbf{\Delta_{j\theta}}}\frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}\right)u'(q)-\alpha'(q)\right]\frac{\partial q}{\partial r}-p_j\frac{\partial d_\theta(P,q)}{\partial \mathbf{p_j}}\frac{\partial \mathbf{p_j}}{\partial r}\end{aligned}$$

The first term is always positive since

$$\left((1-d_\theta(P,q))-p_j\frac{\partial d_\theta(P,q)}{\partial \mathbf{\Delta_{j\theta}}}\frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}\right)u'(q) > -r\frac{\partial d_\theta(P,q)}{\partial \mathbf{\Delta_{j\theta}}}\frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}u'(q)=\alpha'(q)$$

Where the last equality comes from the network owner's problem. This inequality means that the net marginal benefit of increasing quality will always be positive at the level of q chosen by the network owner. An increase in $\mathbf{\Delta_{j\theta}}$ is equivalent to a price drop on the part of the un-integrated firms, so $\frac{\partial d_\theta(P,q)}{\partial \mathbf{\Delta_{j\theta}}}$ decreases in quality, and both $\frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}$ and $1-d_\theta(P,q)$ are bounded above by 1. Therefore from the assumptions on $u(\cdot)$ and $\alpha(\cdot)$

$$\lim_{q\to\infty}\left((1-d_\theta(P,q))-p_j\frac{\partial d_\theta(P,q)}{\partial \mathbf{\Delta_{j\theta}}}\frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}\right)u'(q)-\alpha'(q)=-\infty$$

The RHS of eq. (17) is contonuous in r for $r>0$, so $\frac{\partial q}{\partial r}$ is finite for $q>0$, meaning that there must be a value above which the derivative of surplus with regard to r is negative even if quality is monotonic in r. The regulator will not set r above this value.

There is a potential discontinuity in quality in q at $r=0$ which would ensure that the marginal benefit of increasing r would be positive for $r=0$. This discontinuity could result in non-existence of equilibrium if the derivative of welfare is negative for small r. However, examination of eq. (17) shows that q will go to 0 as r goes to 0 but the Inada conditions on $u(\cdot)$ and $\alpha(\cdot)$ ensure that $\frac{\partial q}{\partial r}>0$ and

$$\lim_{q\to 0}\left((1-d_\theta(P,q))-p_j\frac{\partial d_\theta(P,q)}{\partial \mathbf{\Delta_{j\theta}}}\frac{\partial \mathbf{\Delta_{j\theta}}}{\partial u(q)}\right)u'(q)-\alpha'(q)=\infty$$

So the regulator will wish to increase the rental price for small values of r.

If there exists r which maximizes q, from then from lemma 3 $arg\max_r q$ must be some positive number. At that number $\frac{\partial q}{\partial r}=0$ and so $\frac{\partial U}{\partial r}$ will be negative, meaning that the socially optimal rental price must be below the price at which quality is maximized.

■

A.6 Proof of Proposition 4

Taking the derivative of social welfare with regard to r

$$
\begin{aligned}
\frac{\partial U}{\partial r} = \Bigg(\Bigg[&(1-d_\theta(P,q))+\sum_j\Bigg(\int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{j\theta})F(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon\frac{\partial\Delta_{j\theta}}{\partial u(q)}\\
&-\int_{-\infty}^{\infty}\epsilon f(\epsilon)F(\epsilon+\Delta_{j\theta})f(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon\frac{\partial\Delta_{oj}}{\partial u(q)}\Bigg)\\
&+\int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{o\theta})F(\epsilon+\Delta_{oj})^{n}d\epsilon\frac{\partial\Delta_{o\theta}}{\partial u(q)}\\
&+\int_{-\infty}^{\infty}\epsilon f(\epsilon)F(\epsilon+\Delta_{o\theta})nf(\epsilon+\Delta_{oj})F(\epsilon+\Delta_{oj})^{n-1}d\epsilon\frac{\partial\Delta_{oj}}{\partial u(q)}\\
&+\int_{-\infty}^{\infty}(u(q)-\epsilon)f(\epsilon)F(\epsilon-\Delta_{o\theta})nf(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^{n-1}d\epsilon\frac{\partial\Delta_{j\theta}}{\partial u(q)}\\
&+\int_{-\infty}^{\infty}(u(q)-\epsilon)f(\epsilon)f(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^{n}d\epsilon\frac{\partial\Delta_{o\theta}}{\partial u(q)}\Bigg]u'(q)-\alpha'(q)\Bigg)\frac{\partial q}{\partial r}\\
+\Bigg[\sum_j\Bigg(&\int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{j\theta})F(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon\frac{\partial\Delta_{j\theta}}{\partial r}\\
&-\int_{-\infty}^{\infty}\epsilon f(\epsilon)F(\epsilon+\Delta_{j\theta})f(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon\frac{\partial\Delta_{oj}}{\partial r}\Bigg)\\
&+\int_{-\infty}^{\infty}\epsilon f(\epsilon)f(\epsilon+\Delta_{o\theta})F(\epsilon+\Delta_{oj})^{n}d\epsilon\frac{\partial\Delta_{o\theta}}{\partial r}\\
&+\int_{-\infty}^{\infty}\epsilon f(\epsilon)F(\epsilon+\Delta_{o\theta})nf(\epsilon+\Delta_{oj})F(\epsilon+\Delta_{oj})^{n-1}d\epsilon\frac{\partial\Delta_{oj}}{\partial r}\\
&+\int_{-\infty}^{\infty}(u(q)-\epsilon)f(\epsilon)F(\epsilon-\Delta_{o\theta})nf(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta j\theta)^{n-1}d\epsilon\frac{\partial\Delta_{j\theta}}{\partial r}\\
&+\int_{-\infty}^{\infty}(u(q)-\epsilon)f(\epsilon)f(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^{n}d\epsilon\frac{\partial\Delta_{o\theta}}{\partial r}\Bigg]
\end{aligned}
$$

Using the same substitution rule method to collect the integrals as in appendix A.5 and focusing on the first term

$$
\begin{aligned}
\Bigg[(1-d_\theta(P,q))-&\sum_j\int_{-\infty}^{\infty}\epsilon f(\epsilon)F(\epsilon+\Delta_{j\theta})f(\epsilon-\Delta_{oj})F(\epsilon)^{n-1}d\epsilon\frac{\partial\Delta_{oj}}{\partial u(q)}\\
&+\int_{-\infty}^{\infty}\epsilon f(\epsilon)F(\epsilon+\Delta_{o\theta})nf(\epsilon+\Delta_{oj})F(\epsilon+\Delta_{oj})^{n-1}d\epsilon\frac{\partial\Delta_{oj}}{\partial u(q)}\\
&+p_j\int_{-\infty}^{\infty}f(\epsilon)F(\epsilon-\Delta_{o\theta})nf(\epsilon-\Delta_{j\theta})F(\epsilon-\Delta_{j\theta})^{n-1}d\epsilon\frac{\partial\Delta_{j\theta}}{\partial u(q)}\\
&+p_o\int_{-\infty}^{\infty}f(\epsilon)f(\epsilon-\Delta_{o\theta})F(\epsilon-\Delta_{j\theta})^{n}d\epsilon\frac{\partial\Delta_{o\theta}}{\partial u(q)}\Bigg]u'(q)-\alpha'(q)
\end{aligned}
$$

From symmetry

$$\begin{aligned}
&\sum_j \int_{-\infty}^{\infty} \epsilon f(\epsilon) F(\epsilon+\Delta_{j\theta}) f(\epsilon-\Delta_{oj}) F(\epsilon)^{n-1} d\epsilon \\
=& n\int_{-\infty}^{\infty} \epsilon f(\epsilon) F(\epsilon+\Delta_{j\theta}) f(\epsilon-\Delta_{oj}) F(\epsilon)^{n-1} d\epsilon \\
=& n\int_{-\infty}^{\infty} \epsilon^* f(\epsilon^*) F(\epsilon^*+\Delta_{o\theta}) f(\epsilon^*+\Delta_{oj}) F(\epsilon^*+\Delta_{oj})^{n-1} d\epsilon^*
\end{aligned}$$

Where $\epsilon^* = \epsilon - \Delta_{oj}$. We can plug this back into the first term to get

$$\begin{aligned}
&\Bigg[(1-d_\theta(P,q)) + p_j \int_{-\infty}^{\infty} f(\epsilon) F(\epsilon-\Delta_{o\theta}) n f(\epsilon-\Delta_{j\theta}) F(\epsilon-\Delta_{j\theta})^{n-1} d\epsilon \frac{\partial \Delta_{j\theta}}{\partial u(q)} \\
&\qquad + p_o \int_{-\infty}^{\infty} f(\epsilon) f(\epsilon-\Delta_{o\theta}) F(\epsilon-\Delta_{j\theta})^{n} d\epsilon \frac{\partial \Delta_{o\theta}}{\partial u(q)}\Bigg] u'(q) - \alpha'(q) \\
=& \left[(1-d_\theta(P,q)) + p_j \frac{\partial d_j(P,q)}{\partial \Delta_{j\theta}} \frac{\partial \Delta_{j\theta}}{\partial u(q)} + p_o \frac{\partial d_o(P,q)}{\partial \Delta_{o\theta}} \frac{\partial \Delta_{o\theta}}{\partial u(q)}\right] u'(q) - \alpha'(q)
\end{aligned}$$

Repeating the process with the second term, the derivative simplifies to

$$\begin{aligned}
\frac{\partial U}{\partial r} =& \left(\left[(1-d_\theta(P,q)) + p_j \frac{\partial d_j(P,q)}{\partial \Delta_{j\theta}} \frac{\partial \Delta_{j\theta}}{\partial u(q)} + p_o \frac{\partial d_o(P,q)}{\partial \Delta_{o\theta}} \frac{\partial \Delta_{o\theta}}{\partial u(q)}\right] u'(q) - \alpha'(q)\right) \frac{\partial q}{\partial r} \\
&+ \left(p_j \frac{\partial d_j(P,q)}{\partial \Delta_{j\theta}} \frac{\partial \Delta_{j\theta}}{\partial r} + p_o \frac{\partial d_o(P,q)}{\partial \Delta_{o\theta}} \frac{\partial \Delta_{o\theta}}{\partial r}\right)
\end{aligned}$$

If $arg\max_r q$ does not exist, then for $u(\cdot)$ is sufficiently concave and/or $\alpha(\cdot)$ sufficiently convex, the first term, which represents the marginal social benefit of an increase in quality will go to $-\infty$. If $arg\max_r q$ exists, then the same arguments as above imply, but $\frac{\partial q}{\partial r}$ will approach 0 and may even go below 0. Since $s_j^o(P,q) = o(r^x)$ the second term, which represents the lost surplus from changing prices will always remain negative, and the optimal r^{int} will be below $arg\max_r q$ by the same argument as in the proof of proposition 3.

We can decompose the equation defining optimal investment into

$$\begin{aligned}
\alpha'(q) = \Bigg[&(1-d_\theta(P,q)) + np_j \left(-\frac{\partial d_\theta(P,q)}{\partial u(q)} s_j^\theta(P,q) + s_\theta^j(P,q) \frac{\partial d_j(P,q)}{\partial p_j} \frac{\partial p_j}{\partial u(q)}\right) \\
&+ p_o \left(-\frac{\partial d_\theta(P,q)}{\partial u(q)} s_o^\theta(P,q) + s_\theta^o(P,q) \frac{\partial d_o(P,q)}{\partial p_o} \frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\right)\Bigg] u'(q)
\end{aligned}$$

Rearranging Equation (18), equilibrium investment is determined by

$$\begin{aligned}
\alpha'(q) = \Bigg[& nr \left(-\frac{\partial d_\theta(P,q)}{\partial u(q)} s_j^\theta(P,q) + s_\theta^j(P,q) \frac{\partial d_j(P,q)}{\partial p_j} \frac{\partial p_j}{\partial u(q)}\right) + p_o \left(-\frac{\partial d_\theta(P,q)}{\partial u(q)} s_o^\theta(P,q)\right) \\
&- s_o^j(P,q)(p_o - r) \frac{\partial p_j}{\partial u(q)}\Bigg] u'(q)
\end{aligned}$$

Comparing the right hand side of each equation and rearranging slightly, equilibrium investment will be less than the social optimum if, at q^{int}

$$\begin{aligned}&(1-d_\theta(P,q))+n(p_j-r)\left(-\frac{\partial d_\theta(P,q)}{\partial u(q)}s_j^\theta(P,q)\right)\\&>-\frac{\partial d_j(P,q)}{\partial p_j}\frac{\partial p_j}{\partial u(q)}\Big(s_o^j(P,q)(p_o-r)+n(p_j-r))s_\theta^j(P,q)\Big)-s_\theta^o(P,q)\frac{\partial d_o(P,q)}{\partial p_o}\frac{\mathrm{d}p_o}{\mathrm{d}u(q)}\end{aligned}$$

Otherwise it will equal or exceed the social optimum. ■

B Extension of proofs to distributions with a finite lower bound

From the assumption of $f(\cdot)$ common across all options, any lower bound must be common across all options. For the proof of Lemma 1 to work, we only need that the distributions of ϵ draws share a common lower bound. Without loss of generality we can normalize this lower bound to 0.

If the firms share a common lower bound, then if we let $p_\theta = u(q)$, a consumer must have a draw above $p_j - \min p_{-j}$ to have positive probability of choosing option j, otherwise the option with the minimum price will always be more appealing.Thus demand for firm j will be

$$d_j(P,q)=\int_{p_j-\min p_{-j}}^{\infty} f(\epsilon)\prod_{k\neq j}F(\epsilon+\Delta_{jk})d\epsilon \tag{19}$$

Letting $m = -j$ such that $p_m = \min p_{-j}$ we have that $p_j - p_m = \Delta_{mj}$,. When using the substitution rule as in appendix A.1 we will add Δ_{ij} to the lower bound, but $\Delta_{mj}+\Delta_{ij} = \Delta_{mi}$. Additionally, when taking the derivative with regard to p_j or p_m, we will need to consider the change in the bounds of integration, but using the Leibniz rule this means evaluating the interior function at the lower bound and multiplying by 1 or -1 and $\prod_{k\neq j} F(\epsilon + \Delta_{jk})$ will include $F(\epsilon + \Delta_{jm})$, and $\Delta_{mj} + \Delta_{jm} = 0$, so there will be at least one $F(0) = 0$ in the product, meaning that the additional term from including the lower bound will be 0. Therefore all of the equivalences derived above using the substitution rule will remain valid.